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ROYAL AIRCRAFT ESTABLISHMENT

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A MEANS OF ESTIMATING THE ERRORS IN THE ACCELEROMETERS TO BE USED IN THE BLACK KNIGHT INERTIA TABLE TRIALS

by

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MARCH, 1961

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A MEANS OF ESTIMATING THE ERRORS IN THE ACCELEROMETERS TO BE USED IN THE BLACK KNIGHT INERTIA TABLE TRIALS

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SUMMARY

The method is based on a comparison between the results of double integration of accelerometer output and positions as observed by ground based instrumentation. The effects of errors in ancillary equipment are discussed and tables of accelerometer errors have been computed for a typical Black Knight trajectory. It appears that the accuracy of the ground equipment at present available at Woomera is barely adequate to meet the requirements of the experiment.

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1 INTRODUCTION

The difference between true position and that found by double integration of accelerometer output may be expressed as a function of accelerometer misalignment and platform drift. An over-determined set of equations, based on this relationship between integrated values of recorded acceleration and position observed by ground instrumentation for various times during flight, gives a means of evaluating errors in the accelerometers themselves. Further, if the r.m.s. errors of observation are known it is possible to predict the accuracy with which the accelerometer errors may be determined.

This has been done, using a basic Black Knight acceleration curve. An estimate of the accuracy with which the accelerometer bias and scale errors can be calculated is made and tables have been drawn up showing the weights which should be given to the probable experimental errors, both random and systematic, in the ancillary equipment.

2 COMPONENTS OF ERROR IN THE ACCELEROMETERS

Errors which are anticipated in the accelerometers are given below with the notation to be used subsequently.

(1) Bias k_0.

(2) Constant scale error k_1.

(3) Scale error varying with acceleration $k_2\ddot{x}$, where k_2 is constant.

(4) Cross coupling errors.

(5) Misalignment errors.

For the Kearfott accelerometers with which the equipment is concerned initially, the expected values are

$$k_0 \simeq 2 \times 10^{-4} g \simeq 6.4 \times 10^{-3} \text{ ft sec}^{-2}$$

$$k_1 \simeq 2 \times 10^{-4}$$

$$k_2 \simeq 2 \times 10^{-7} \text{ ft}^{-1} \text{ sec}^2 \quad .$$

The cross coupling error when measured in laboratory tests was small compared with these and has been ignored. In theory the axes of the accelerometers will coincide with the fixed space axes defined at launch. In practice there will be errors in lining them up but the discrepancies will be measured and should be known to 5 seconds of arc.

3 NOTATION

The axes used are fixed space axes and defined as O,XYZ where O is the centre of the table and OZ is the local vertical at launch, OY the direction of the line of fire and OX completes the right handed set of orthogonal axes. It will be assumed that the accelerometers are at the centre of gravity of the missile; also that all positions determined from ground instrumentation will have been corrected for the rotation and curvature of the earth and refer to the fixed space axes.

$\ddot{x}$	true acceleration in the direction OX at time t
$\ddot{x}_{acc.}$	acceleration experienced by the x accelerometer at time t
$\ddot{x}_r$	recorded accelerometer output from the x accelerometer at time t
g_{xt}, g_{yt}, g_{zt}	gravity components at time t
x_{gt}, y_{gt}, z_{gt}	position from ground instrumentation at time t
θ_{xx}, $\frac{\pi}{2} - \theta_{xy}$, $\frac{\pi}{2} - \theta_{xz}$	angles between the axis of the x accelerometer and OX, OY, OZ at launch where θ is small
ϕ_{xt}, ϕ_{yt}, ϕ_{zt}	the components of table drift about the fixed axes O,XYZ at time t. ϕ small
ℓ_x, m_x, n_x; ℓ_y, m_y, n_y; ℓ_z, m_z, n_z	the direction cosines of the accelerometer axes at launch so that $\ell_x \simeq 1$ $m_x \simeq \theta_{xy}$ $n_x \simeq \theta_{xz}$
k_{0x}	bias error in the x accelerometer
k_{1x}	constant scale error in the x accelerometer
$k_{2x}\ddot{x}$	scale error varying with acceleration, k_2 constant
k_{xt}	$\equiv \dfrac{k_{0x} + k_{1x}\ddot{x}_{acc.} + k_{2x}\ddot{x}_{acc.}^2}{\ddot{x}_{acc.}}$
s_{xT}, v_{xT}	position and velocity error components along the x axis at time T
τ	a sampling interval
A_{mn}	a rectangular matrix of dimensions m $\times$ n
A_m	a diagonal matrix of dimensions m
a_m	a column vector of dimension m
a_m^*	a row vector of dimension m
α, β	the angular displacements of a star in the field of view of a television camera
ℓ_c, m_c, n_c	the direction cosines of the line of sight of a star relative to the camera axes

f the focal length of the camera

ε a random error

$$\int_{T}^{T+t'} \ddot{x} dt^2 \equiv \int_{T}^{T+t'} du \int_{T}^{u} \ddot{x}(v)\, dv$$

4 METHOD OF EVALUATING THE COMPONENTS OF ERROR IN THE ACCELEROMETERS

The method described for determining errors in the accelerometers has been developed from that suggested by Chester[1] which was found to be impracticable when applied to a standard Black Knight trajectory during thrust. All accelerometer errors, drift angles and misalignment angles are supposed small and second order terms are neglected.

At time t the direction cosines of the X accelerometer axis with respect to O,XYZ will be

$$(\ell_x - \phi_{zt} m_x + \phi_{yt} n_x \quad , \quad \phi_{zt}\ell_x + m_x - \phi_{xt} n_x \quad , \quad -\phi_{yt}\ell_x + \phi_{xt} m_x + n_x)$$

or to the first order.

$(1, \ \phi_{zt} + \theta_{xy}\ , \ -\phi_{yt} + \theta_{xz})$ and the acceleration experienced by the accelerometer will be

$$\ddot{x}_{acc.} = \ddot{x} + g_x + (\phi_{zt} + \theta_{xy})(\ddot{y} + g_y) + (-\phi_{yt} + \theta_{xz})(\ddot{z} + g_z) \ .$$

Now

$$\ddot{x}_r = \ddot{x}_{acc.} + k_{0x} + k_{1x}\ddot{x}_{acc.} + k_{2x}\ddot{x}_{acc.}^2$$

and this can be expressed as

$$\ddot{x}_r = \ddot{x}_{acc.} + k_{xt}\ddot{x}_{acc.}$$

where

$$k_{xt} = \frac{k_{0x} + k_{1x}\ddot{x}_{acc.} + k_{2x}\ddot{x}_{acc.}^2}{\ddot{x}_{acc.}} \ .$$

Hence, at time t, to the first order

$$\ddot{x}_r = \ddot{x} + g_x + (\phi_{zt} + \theta_{xy})(\ddot{y} + g_y) + (-\phi_{yt} + \theta_{xz})(\ddot{z} + g_z) + k_{xt}(\ddot{x} + g_x) \quad \dots\dots (1)$$

Now if x_T and $\dot{x}_T$ are the true position and velocity at a time T, the position at any other time T+t' can be written

$$x_{T+t'} = x_T + \dot{x}_T t' + \int_T^{T+t'} \ddot{x} dt^2 \qquad (2)$$

and similarly

$$x_{r,T+t'} = x_{r,T} + \dot{x}_{r,T} t' + \int_T^{T+t'} \ddot{x}_r dt^2 \;. \qquad (3)$$

But, because of drifts and errors

$$x_{r,T} = x_T + s_{xT}$$

and

$$\dot{x}_{rT} = \dot{x}_T + v_{xT}$$

where s_{xT}, v_{xT} are error terms, and equation (3) becomes

$$x_{r,T+t'} = x_T + s_{xT} + \dot{x}_T t' + v_{xT} t'$$

$$+ \int_T^{T+t'} [\ddot{x}+g_x+(\phi_{zt}+\theta_{xy})(\ddot{y}+g_y)+(-\phi_{yt}+\theta_{xz})(\ddot{z}+g_z)+k_{xt}(\ddot{x}+g_x)]\, dt^2 \;. \qquad (4)$$

Subtracting (2) from (4)

$$x_{r,T+t'} - x_{T+t'} = s_{xT} + v_{xT} t' + \int_T^{T+t'} g_x\, dt^2 + \int_T^{T+t'} (\phi_{zt}+\theta_{xy})\,(\ddot{y}+g_y)\, dt^2$$

$$+ \int_T^{T+t'} (-\phi_{yt}+\theta_{xz})\,(\ddot{z} + g_z)\, dt^2 + \int_T^{T+t'} k_{xt}\,(\ddot{x} + g_x)\, dt^2 \;. \qquad (5)$$

Rewriting this

$$s_{xT} + v_{xT} t' + \int_T^{T+t'} k_{xt}(\ddot{x}+g_x) dt^2 + \int_T^{T+t'} \phi_{zt}(\ddot{y}+g_y) dt^2 + \int_T^{T+t'} (-\phi_{yt})\,(\ddot{z} + g_z)\, dt^2$$

$$= x_{r,T+t'} - x_{T+t'} - \int_T^{T+t'} g_x\, dt^2 - \int_T^{T+t'} \theta_{xy}(\ddot{y}+g_y)\, dt^2 - \int_T^{T+t'} \theta_{xz}(\ddot{z}+g_z)\, dt^2 \qquad (6)$$

and putting $x_g = x + \varepsilon$, to the first order this is equivalent to

$$s_{xT} + v_{xT}t' + \int_T^{T+t'} k_{xt}\ddot{x}_r dt^2 + \int_T^{T+t'} \phi_{zt}\ddot{y}_r dt^2 + \int_T^{T+t'} (-\phi_{yt}\ddot{z}_r)\, dt^2$$

$$= x_{r,T+t'} - x_{g,T+t'} - \int_T^{T+t'} g_x dt^2 - \int_T^{T+t'} \theta_{xy}\ddot{y}_r dt^2 - \int_T^{T+t'} \theta_{xz}\,\ddot{z}_r\, dt^2 + \varepsilon_{t'} \;. \qquad (7)$$

The following is Chester's[1] approach. Consider a sampling period $T-n\tau$ to $T+n\tau$ containing $(2n+1)$ samples at equal intervals τ. For each sample there is an equation (7). Define s_x, v_x, k_x, ϕ_z, ϕ_y as those constants whioh when replacing s_{xT}, v_{xT}, k_{xt}, ϕ_{zt}, ϕ_{yt} in equations (7) reduce $\sum \varepsilon_{t'}^2$ over all equations to a minimum. Then if $\varepsilon_{t'}$ is random, by the least squares criterion, these will be the 'best' estimates of the values of k_{xT}, ϕ_{zT} and ϕ_{yT}. For a missile constrained to fly along a straight line near the vertical this method breaks down. Rewriting equation (6) with the constants we have

$$s_x + v_x t' + k_x \int_T^{T+t'} (\ddot{x}+g_x)dt^2 + \phi_z \int_T^{T+t'} (\ddot{y}+g_y)dt^2 - \phi_y \int_T^{T+t'} (\ddot{z}+g_z)dt^2 = f(t') + \varepsilon_{t'}$$

but now

$$\frac{\int_T^{T+t'} (\ddot{z} + g_z)\, dt^2}{\int_T^{T+t'} (\ddot{x} + g_x)\, dt^2}$$

remains almost constant over short sampling periods and similarly

$$\frac{\int_T^{T+t'} (\ddot{y} + g_y)\, dt^2}{\int_T^{T+t'} (\ddot{x} + g_x)\, dt^2} ,$$

and the equation approximates to the form

$$s_x + v_x t' + (k_x + c_1\phi_z - c_2\phi_y) \int_T^{T+t'} (\ddot{x} + g_x)\, dt^2 = f(t') + \varepsilon_{t'}$$

where o_1 and o_2 are constants. Hence for motion in a near vertical straight line the values of k_x, ϕ_z and ϕ_y are not separable and the acceleration errors can only be determined if the drift is known.

Provision has been made in the Black Knight inertia table experiment to measure drift. Hence equation (6) may be expressed in the form

$$s_{xT} + v_{xT}t' + \int_T^{T+t'} k_{xt}(\ddot{x}+g_x)dt^2 = x_{r,T+t'} - x_{g,T+t'} - \int_T^{T+t'} g_{xt}dt^2$$

$$- \int_T^{T+t'} (\phi_{zt}+\theta_{xy})(\ddot{y}+g_y)dt^2 - \int_T^{T+t'} (-\phi_{yt}+\theta_{xt})(\ddot{z}+g_z)dt^2 + \varepsilon_{t'} \qquad (6)$$

where the quantities on the right hand side are measurable and can be sampled at equal time intervals. Replacing the integrals by the appropriate summations we have

$$s_{xT} + v_{xT}(i\tau) + \tau^2 \sum_{k=0}^{i} \sum_{j=0}^{k} k_{xj}(\ddot{x}_{T+j\tau} + g_{x,T+j\tau})$$

$$= x_{r,T+i\tau} - x_{g,T+i\tau} - \tau^2 \sum_{k=0}^{i} \sum_{j=0}^{k} g_{x,T+j\tau} - \tau^2 \sum_{k=0}^{i} \sum_{j=0}^{k} (\phi_{z,T+j\tau}+\theta_{xy})(\ddot{y}_{T+j\tau}+g_{y,T+j\tau})$$

$$- \tau^2 \sum_{k=0}^{i} \sum_{j=0}^{k} (-\phi_{y,T+j\tau}+\theta_{xz})(\ddot{z}_{T+j\tau} + g_{z,T+j\tau}) + \varepsilon'_{i\tau} \qquad . \qquad (8)$$

Two ways are open from here. Either k_{xj} can be replaced by a k_x, without physical counterpart, which is assumed to be constant throughout any one sampling period and whose value computed over all samples is used as the value of k_{xT} at the centre of the period, or it may be split into its components k_{0x} k_{1x}, k_{2x} ..., which are constant throughout flight, and the trajeotory from launch to burn out considered as a single sampling period. Additional information is then available, in that both velocity and position are known to be zero when $t' = -n\tau$, and two rigorous equations must be solved with the modified normal equations, using the method of Lagrangian undetermined multipliers. When k_{0x}, k_{1x}, k_{2x} are determined directly, the computing is very much simpler when done from launch as s and v are then zero and need not be estimated. But

if the sampling rate is high the numbers involved are immense, and it is possible for rounding errors in the calculation of $\sum x^2 \sum t^4 - \left(\sum xt^2\right)$ to invalidate the answers. Using the first method, by taking a succession of sampling periods throughout the flight and fitting a polynomial $k_{0x} + k_{1x}\ddot{x}_{acc.} + k_{2x}\ddot{x}_{acc.}^2 + \ldots$ to the values of $k_{xT}\ddot{x}_{acc.}$ for the centre of each period, the components of accelerometer error may be estimated. Where the number of components is uncertain this form has the advantage that the degree of the polynomial finally fitted is easily adjusted.

5 ANALYSIS OF ERRORS DUE TO ANCILLARY INSTRUMENTATION AND METHOD OF COMPUTING

5.1 Errors in k_{0x}, k_{1x}, k_{2x} in terms of the errors in k_{xT}.

By definition

$$x_{acc.,T} \times k_{xT} = k_{0x} + k_{1x} x_{acc.,T} + k_{2x} x_{acc.,T}^2$$

for each T. But the estimates of k_{xT} found in the experiment will not be without error. Call the error ε_T. Then if there are n sampling periods, there will be n equations

$$\begin{bmatrix} \frac{1}{\ddot{x}_{acc.,1}} & 1 & \ddot{x}_{acc.,1} \\ \frac{1}{\ddot{x}_{acc.,2}} & 1 & \ddot{x}_{acc.,2} \\ & \vdots & \\ \frac{1}{\ddot{x}_{acc.,n}} & 1 & \ddot{x}_{acc.,n} \end{bmatrix} \begin{bmatrix} k_{0x} \\ k_{1x} \\ k_{2x} \end{bmatrix} = \begin{bmatrix} k_{x1} + \varepsilon_1 \\ k_{x2} + \varepsilon_2 \\ \vdots \\ k_{xn} + \varepsilon_n \end{bmatrix}$$

where k_{xi} is now the experimental value.

Writing this in matrix form as

$$A_n \begin{bmatrix} k_{0x} \\ k_{1x} \\ k_{2x} \end{bmatrix} = K_n$$

then, solving by least squares for the best estimates of k_{0x}, k_{1x}, k_{2x},

$$A^*A \begin{bmatrix} k_{0x} \\ k_{1x} \\ k_{2x} \end{bmatrix} = A^*K_n \quad .$$

The error equations

$$A^*A \begin{bmatrix} \delta k_{0x} \\ \delta k_{1x} \\ \delta k_{2x} \end{bmatrix} = \overline{A^*\delta K_n}$$

follow, and the variances of k_{0x}, k_{1x}, k_{2x} in terms of the variance of k_{xT} can be calculated, with the proviso that if the variance of k_{xT} changes systematically through the trajectory it will be necessary to weight the equations.

5.2 Sources of error in k_x

The equations (8) for chosen values of T may be written in matrix form for a period of m(=2n+1) samples as

$$A_{m\times 3}\, q_3 = b_m + c_m + d_m + e_m$$

where q_3 is the vector $\begin{bmatrix} s_{xT} \\ v_{xT} \\ k_{xT} \end{bmatrix}$ and

$$b_m = \Big[x_{r,T+i\tau} \Big]_m$$

$$c_m = \Big[-x_{g,T+i\tau} - \tau^2 \sum_{k=0}^{i} \sum_{j=0}^{k} g_{x,T+j\tau} \Big]_m$$

$$d_m = \Big[-\tau^2 \sum_{k=0}^{i} \sum_{j=0}^{k} (\phi_{z,T+j\tau} + \theta_{xy})\, (\ddot{y}_{T+j\tau} + g_{y,T+j\tau}) \Big]_m$$

$$e_m = \Big[-\tau^2 \sum_{k=0}^{i} \sum_{j=0}^{k} (-\phi_{y,T+j\tau} + \theta_{xz})\, (\ddot{z}_{T+j\tau} + g_{z,T+j\tau}) \Big]_m \quad .$$

The normal equations are then

$$A^*Aq_3 = A^*[b_m + c_m + d_m + e_m]$$

and the most probable value of k_{xT} is the third element of q_3 in the solution

$$q_3 = [A^*A]^{-1}A^*[b_m + c_m + d_m + e_m] \quad .$$

If δq_3 is the error in q_3 due to small errors δb_m, δc_m, δd_m, δe_m in b_m, c_m d_m, e_m

$$\delta q_3 = [A^*A]^{-1} A^* [\delta b_m + \delta c_m + \delta d_m + \delta e_m] \quad ,$$

since the elements of A are constant. In particular, if p_m^* is the last row in the matrix $[A^*A]^{-1}A^*$

$$\delta k_{xT} = p_m^* \left[\delta b_m + \delta c_m + \delta d_m + \delta e_m \right] \quad .$$

Since the sources of the errors in b_m, c_m, d_m and e_m are independent

$$E(\delta k_{xT})^2 = E(p_m^* \delta b_m)^2 + E(p_m^* \delta c_m)^2 + E(p_m^* \delta d_m)^2 + E(p_m^* \delta e_m)^2 \quad ,$$

and the contributions to the r.m.s. error in k_{xT} from the various sources may be considered separately.

5.3 Errors in recording the data

Details of errors inherent in the different methods of recording the data are given, with estimates of the variances, as appendices. In the main they fall into 3 classes

(1) constant,

(2) random,

(3) slowly varying cyclic.

Consider the errors from source 'b' only. If the vector p_m^* has elements p_i, $-n \leqslant i \leqslant n$

$$\delta k_x = \sum_i p_i \, \delta b_i \tag{9}$$

$$(\delta k_x)^2 = \left(\sum_i p_i \, \delta b_i \right)^2$$

$$E(\delta k_x)^2 = E\left(\sum_i p_i^2\,\delta b_i^2 + 2\sum_i\sum_j p_i p_j\,\delta b_i\,\delta b_j\right) \qquad i < j$$

$$= \sum_i p_i^2\,E(\delta b_i^2) + 2\sum_i\sum_j p_i p_j\,E(\delta b_i\,\delta b_j)\ .$$

(1) If δb_i is constant and equal to σ_b

$$E(\delta k_x)^2 = \left(\sum p_i^2 + 2\sum_i\sum_j p_i p_j\right)\sigma_b^2$$

$$= \left(\sum p_i\right)^2 \sigma_b^2\ .$$

(2) If δb_i is random with variance σ_b^2

$$E(\delta k_x)^2 = \sum_i p_i^2\,\sigma_b^2$$

since

$$E(\delta b_i\,\delta b_j) = 0\ .$$

(3) If δb_i is slowly varying and cyclic, let $\delta b_i = a\sin(\omega t_i + \phi)$ where ϕ in repeated trials is random.

Then

$$E(\delta k_x)^2 = \sum_i p_i^2 a^2\,E(\sin^2(\omega t_i+\phi)) + 2a^2\sum_i\sum_j p_i p_j E(\sin(\omega t_i+\phi)\sin(\omega t_j+\phi))$$

$$= \tfrac{1}{2}a^2\sum_i p_i^2\,E(1-\cos 2(\omega t_i+\phi)) + a^2\sum_i\sum_j p_i p_j E\Big[\cos\omega(t_i-t_j) - \cos[\omega(t_i+t_j)+2\phi]\Big]$$

$$= \tfrac{1}{2}a^2\sum_i p_i^2 + a^2\sum_i\sum_j p_i\,p_j\,\theta \qquad -1 < \theta < 1\ .$$

Now if a is the amplitude of the oscillation $\frac{1}{2}a^2$ is the variance. So for a slow cyclic variation with random phase

$$E(\delta k_x)^2$$

lies between

$$\left(\sum_i p_i^2 - 2\sum_i\sum_j p_i p_j\right)\sigma_b^2$$

and

$$\left(\sum_i p_i^2 + 2\sum_i\sum_j p_i p_j\right)\sigma_b^2 \quad .$$

5.3.1 Errors in position due to errors in recording acceleration and their effect on k_x

Taking a general case, suppose that at time t an acceleration is $\ddot{x}$ and that the error in the recorded acceleration is $\varepsilon\,\ddot{x}$, then $\int_T^{T+t'} \varepsilon\ddot{x}dt^2$ may be written as $\int_0^{t'} \varepsilon\ddot{x}(t'-q)dq$ and putting this in the form of a summation for sampled data using the trapezoidal rule

$$\int_T^{T+t'} \varepsilon\ddot{x}\,dt^2 \;\simeq\; \pm\,\tau^2\left\{\frac{\ddot{x}_o}{2} + \sum_{i=1}^{m} \varepsilon_i\,\ddot{x}_i(m-i)\right\} \quad ,$$

where τ is the sampling interval, $\varepsilon_i\ddot{x}_i = \varepsilon\ddot{x}_{T+i\tau}$, $m\tau = t'$ and the sign is the sign of m. If δk_ε is the error in k_x due to errors $\varepsilon\ddot{x}$ only, then it follows from (9) that

/Equation (10)

$$
\begin{aligned}
\frac{\delta k_\varepsilon}{\tau^2} = \; & p_{-n}\left(n\varepsilon_o \frac{\ddot{x}_o}{2} + (n-1)\,\varepsilon_{-1}\ddot{x}_{-1} + \dots + \varepsilon_{-(n-1)}\ddot{x}_{-(n-1)}\right) \\
& + p_{-(n-1)}\left((n-1)\,\varepsilon_o \frac{\ddot{x}_o}{2} + (n-2)\,\varepsilon_{-1}\,\ddot{x}_{-1} + \dots + \varepsilon_{-(n-2)}\ddot{x}_{-(n-2)}\right) \\
& + \\
& \vdots \\
& + p_{-2}\left(2\varepsilon_o \frac{\ddot{x}_o}{2} + \varepsilon_{-1}\,\ddot{x}_{-1}\right) \\
& + p_{-1}\left(\varepsilon_o \frac{\ddot{x}_o}{2}\right) \\
& + p_o \times 0 \\
& + p_1\left(\varepsilon_o \frac{\ddot{x}_o}{2}\right) \\
& + p_2\left(2\,\varepsilon_o \frac{\ddot{x}_o}{2} + \varepsilon_1\ddot{x}_1\right) \\
& + \\
& \vdots \\
& + p_n\left(n\varepsilon_o \frac{\ddot{x}_o}{2} + (n-1)\,\varepsilon_1\ddot{x}_1 + \dots + \varepsilon_{n-1}\ddot{x}_{n-1}\right)
\end{aligned}
\qquad (10)
$$

$$
\begin{aligned}
= \; & \varepsilon_{-n}\,\ddot{x}_{-n}\left[p_{-n} \times 0\right] \\
& + \varepsilon_{-(n-1)}\,\ddot{x}_{-(n-1)}\left[p_{-n} \times 1\right] \\
& + \varepsilon_{-(n-2)}\,\ddot{x}_{-(n-2)}\left[2p_{-n} + p_{-(n-1)}\right] \\
& + \\
& \vdots \\
& + \varepsilon_o \frac{\ddot{x}_o}{2}\left[np_{-n} + (n-1)\,p_{-(n-1)} + \dots + 2p_{-2} + p_{-1}\right] \\
& + \varepsilon_o \frac{\ddot{x}_o}{2}\left[np_n + (n-1)\,p_{n-1} + \dots + 2p_2 + p_1\right] \\
& + \\
& \vdots \\
& + \varepsilon_{n-2}\,\ddot{x}_{n-2}\left[p_n \times 2 + p_{n-1} \times 1\right] \\
& + \varepsilon_{n-1}\,\ddot{x}_{n-1}\left[p_n \times 1\right] \\
& + \varepsilon_n\,\ddot{x}_n\left[p_n \times 0\right]
\end{aligned}
\qquad (11)
$$

and δk_ε may now be written

$$
\delta k_\varepsilon = \tau^2 \sum_i q_i \qquad -n < i < n
$$

where

$$q_i = \ddot{x}_i \left[[(n+i)\ p_{-n} + [(n-1)+i]\ p_{-(n-1)} + \dots\ ] \right] \qquad -n \leq i < 0$$

$$q_o = \ddot{x}_o \left[\frac{n}{2} (p_{-n} + p_n) + \frac{n-1}{2} (p_{-(n-1)} + p_{(n-1)} + \dots + \tfrac{1}{2}(p_{-1} + p_{+1}) \right]$$

$$q_i = \ddot{x}_i \left[(n-i)\ p_n + (n-1-i)\ p_{n-1} + \dots \right] \quad . \qquad 0 < i \leq n$$

5.4 Contributions to the variance of k_x from different sources

5.4.1 Bias error in recorded acceleration used to calculate x_r

This follows from the previous paragraph if $\ddot{x}_i$ is made equal to unity so that the error $\varepsilon\ddot{x}_i$ is independent of $\ddot{x}$. If σ^2 is the expected variance of the bias errors, the contribution to the variance of δk_x is $\sigma^2 \sum_i q_i^2$ $\sigma^2 \left(\sum_i q_i\right)^2$ or between $\sigma^2 \left(\sum q_i^2 \pm 2\sum_i \sum_j q_i q_j\right)$ according to whether the errors are random, constant, or correlated.

5.4.2 Scale errors in the recorded accelerometer output

This is the case in para. 5.3.1 when $\ddot{x}$ is replaced by $\ddot{x}_{acc.}$, and ε represents the scale factor.

5.4.3 Errors in position from ground instrumentation

These are independent of acceleration and can be expressed directly in terms of the p_s as in para. 5.3. If σ^2 is their variance the contribution is

$$\sigma^2 \sum_i p_i^2 \qquad \text{where the errors are random}$$

$$\sigma^2 \left(\sum_i p_i\right)^2 \qquad \text{for constant errors}$$

and between

$\sigma^2 \left(\sum_i p_i^2 - 2 \sum_i \sum_j p_i p_j\right)$ and $\sigma^2 \left(2p_i^2 + \sum \sum p_i p_j\right)$ if the errors are correlated. Errors in position which are systematic and can be expressed as a linear function of time during the sampling period are immaterial. For, in

the equations (6) from which k_{xT} is calculated, if $x_{g,T+t'} = x_{T+t'}+\Delta x+\Delta vt'$,

$$s_{xT} + v_{xT}t' + k_{xT}\int_{t'}^{T+t'} (\ddot{x} + g_x)dt^2 = x_{r,T+t'} - x_{g,T+t'} + \text{other terms}$$

can be written

$$(s_{xT} + \Delta x) + (v_{xT} + \Delta v)t' + k_{xT}\int_{T'}^{T+t'} (\ddot{x} + g_x)dt^2$$

$$= x_{r,T+t'} - x_{T+t'} + \text{other terms},$$

showing that the errors Δx and Δv are absorbed by s_{xT} and v_{xT} in the least squares solution.

5.4.4 Errors in drift angle measurement

Since the drift angle enters the equations as a factor multiplying acceleration, the method of computing the error contribution to $E(\delta k_x)^2$ follows that used for the acceleration scale error. But whereas for all other sources it has been assumed that there is an observation with its associated error for each sampling instant the method of measuring drift restricts the data to one sample per second. If a faster sampling rate is used intermediate drift angles must be found by interpolation and will be correlated in groups. A simple example to show the method of computing is given below, where $n = 4$, $\tau = \frac{1}{2}$ and $\varepsilon_i = \dfrac{\varepsilon_{i-1} + \varepsilon_{i+1}}{2}$ for i odd.

From equation (11)

$$\begin{aligned}
\frac{\delta k_\varepsilon}{\tau^2} = {} & \varepsilon_{-4}\,\ddot{x}_{-4}\,(p_{-4} \times 0) \\
& + \frac{(\varepsilon_{-4} + \varepsilon_{-2})}{2}\,\ddot{x}_{-3}(1 \times p_{-4}) \\
& + \varepsilon_2\,\ddot{x}_{-2}\,(2p_{-4} + p_{-3}) \\
& + \frac{(\varepsilon_2 + \varepsilon_0)}{2}\,\ddot{x}_{-1}\,(3p_{-4} + 2p_{-3} + p_{-2}) \\
& + \varepsilon_0\,\frac{\ddot{x}_0}{2}\left[4(p_{-4}+p_4)+3(p_{-3}+p_3)+2(p_{-n}+p_2)+(p_{-1}+p_1)\right] \\
& + \frac{(\varepsilon_0 + \varepsilon_2)}{2}\,\ddot{x}_1\,(3p_4 + 2p_3 + p_2) \\
& + \varepsilon_2\,\ddot{x}_2\,(2p_4 + p_3) \\
& + \frac{(\varepsilon_2 + \varepsilon_4)}{2}\,\ddot{x}_3\,p_4 \\
& + \varepsilon_4\,\ddot{x}_4(0) \quad .
\end{aligned}$$

This is rewritten as

$$\frac{\delta k_\varepsilon}{\tau^2} = \varepsilon_{-4}\left[\ddot{x}_{-4}\times 0 + \frac{\ddot{x}_{-3}}{2}p_{-4}\right] + \varepsilon_{-2}\left[\frac{\ddot{x}_{-3}}{2}p_{-4} + \ddot{x}_{-2}\,(2p_{-4}+p_{-3}) + \frac{\ddot{x}_{-1}}{2}(3p_{-4}+2p_{-3}+p_{-2})\right] + \varepsilon_0\left[\quad\right] + \ldots.$$

i.e.

$$\delta k_\varepsilon = \tau^2 \sum_i r_{2i}\,\varepsilon_{2i} \qquad -\frac{n}{2} < i < \frac{n}{2}$$

and the standard deviation of ε_{2i} , i.e. of the observations, can still be used, but with different weights.

5.4.5 Accelerometer misalignment errors

Since the misalignment will be measured before launch it is only the uncertainty in the measurements which need be considered. This will give constant errors on any one trial and the contribution to $E(\delta k_x)^2$ will be given by $\sigma^2\left(\sum_i q_i\right)^2$ where σ is the r.m.s. error of measurement and $\ddot{x}$ is replaced by $\ddot{y}_{acc.}$ for θ_{xy} and by $\ddot{z}_{acc.}$ for θ_{xz}.

5.4.6 The error in using a constant 'k' throughout the interval as an estimate of the value at the centre of the interval

When the acceleration can be expressed as a linear function of time; let $(\ddot{x}+g_x) = a + bt$ where t is the time from the centre of the interval, T. (It is necessary that the length of the interval is such that for maximum t, $bt/a < 1$.) Then

$$k_{x,T+t} = \frac{k_{0x}}{a+bt} + k_{1x} + k_{2x}(a+bt)$$

and

$$k_{xT} = \frac{k_{0x}}{a} + k_{1x} + k_{2x}a$$

at the centre of the interval. In each equation (6) the true value of $k_{x,T+t}$ can be written as

$$\frac{k_{0x}}{a}\left(1 - \frac{bt}{a} + \left(\frac{b}{a}\right)^2 t^2 - \ldots\right) + k_1 + k_2 a + k_2 bt \quad .$$

Thus the error introduced into each equation by taking $k_{x,T+t}$ at the central value is

$$\int_{T}^{T+t'} \left[\frac{k_{0x}}{a}\left(-\frac{b}{a}t + \left(\frac{b}{a}\right)^2 t^2 - \ldots\right)(a+bt) + k_{2x}bt(a+bt)\right] dt^2$$

$$= k_{0x}\int_{T}^{T+t'} -\frac{b}{a}t\,dt^2 + k_{2x}\int_{T}^{T+t'} bt(a+bt)\,dt^2 .$$

Since k_{0x}, k_{2x}, although unknown, are constants the variance of the error in k_{xT} is given by

$$\left[k_0^2\left(\sum_i p_i\right)^2 + k_2^2\left(\sum_i q_i\right)^2\right]$$

where the p_i's are the elements of the last row of the matrix $(A^*A)^{-1}\,A^*N$

where N is the vector $\left[\int_{T}^{T+t'} -\frac{b}{a}t\,dt^2\right]$ and the q_i's the corresponding terms of $(A^*A)^{-1}\,A^*M$ where M is the vector $\left[\int_{T}^{T+t'} bt(a+bt)dt^2\right]$.

6 ERROR COMPUTATION FOR AN APPROPRIATE BLACK KNIGHT TRAJECTORY

All the relationships which have been developed for the X co-ordinate apply equally well to Y and Z. The error equations have been programmed for Pegasus and used to estimate the accuracy with which it might be hoped to assess the component errors in the accelerometers in the inertia table trials. A trajectory, assumed to be a straight line at 2° to the vertical was computed against time from launch from typical B.K. accelerations. Approximations to the acceleration function were also made by fitting straight lines centred on 10 secs, 30 secs ..... to 130 secs and the simpler form used within any one sampling period for the method of computation where k_{xT} was assumed constant.

The tables 1a and 1b which follow show the weights to be applied to the variances of errors from trials instrumentation for different sampling rates and sampling periods and the corresponding $E(\delta k)^2$. Tables 2a and 2b show the variances of the errors in k_0, k_1, k_2 in terms of that of k and also those of k_0 and k_1 when k_2 is absent.

The variances of k_0 and k_1, and k_0, k_1 and k_2 calculated directly over the whole flight are given in tables 3a and 3b. These were computed by integrating from launch instead of from the centre of the thrusting period. When the data are considered as a whole the question of weighting equations (8) becomes more important. But the choice of weights is difficult as the differences

between the positions from ground instrumentation and those estimated by integrating the accelerometer outputs are due to the combination of various errors requiring different weighting factors. For the Black Knight experiment the predominant cause of error in k_{0z}, k_{1z}, k_{2z} was found from the earlier calculations to lie in the ground instrumentation. For this only, extra values are given in table 3a for the z co-ordinate showing the reduction in the variances of the k's which would result from weighting the equations, were this considered feasible. The question also arises, what errors would be made in k_0 and k_1 if k_2 were not zero, but ignored? These are shown in tables 4.

6.1 Results for the Z accelerometer

Calculation of k_{zT}, followed by k_{0z}, k_{1z}, k_{2z}. By taking a period of 30 seconds and sampling at $\frac{1}{2}$ second intervals the r.m.s. error in k_{zT} can be reduced to the order of 2×10^{-4} and with 10 such periods and no k_{2z} term present, the error in k_{0x} is of the order of 1.3×10^{-2}g and in k_{1z} 1.3×10^{-4}. It is unlikely that these values could be improved much by increasing either the length or number of sampling periods as the nature of the instrumentation errors is such that correlation would arise between intermingled samples. So without k_{2z} the accuracy of determination of k_{1z} by this method is hardly good enough and for k_{0z} it is not good enough. The plot of $k_z \ddot{z}_r$ versus $\ddot{z}_r$ should however indicate whether or not a k_{2z} term is to be expected and to this extent the computation would be of some value. With k_{2z} present the accuracy is too low to assess either k_{0z} or k_{1z} satisfactorily.

The results of the second method, whereby k_{0z}, k_{1z} and k_{2z} are calculated directly by considering the whole time to burn out as a single sampling period are shown in tables 3a for sampling intervals of 5 seconds, 1 second, and 0.2 second. For the z accelerometer this appears to be the better method but the accuracy is still too low to meet the requirements for either k_{0z} or k_{1z} when k_{2z} is determined.

6.2 Results for the Y accelerometer

Using the first method, the errors in k_y are, at best of the order of 1×10^{-2} and the method fails for k_{1y} and k_{2y}. But the error in k_{0y}, 3×10^{-5}g for ten thirty second sampling periods, suggests that it should be possible to detect a zero error in the Y accelerometer and that for this axis the first method is better. For Y, using the second method the error introduced in k_{0y} if k_{1y} is ignored is of the order of $0.7\, k_{1y}$. Hence if the results show k_{1y} to be small the errors in k_{0y} in table 3b might be reduced by ignoring k_{1y} as well as k_{2y} and solving for k_{0y} only.

7 DISCUSSION OF RESULTS

In all the computations made for z the ground instrumentation random errors were predominant. These were finally weighted as for the radar, on the assumption that the F.P.S. 16 would have random errors of 0.04 mils in angle and 8.5 ft in range, giving errors in the z co-ordinate varying from 4 ft at launch to 12 at burnout. Those from ballistic cameras would be approximately half these but, taking a sampling interval of 5 secs, the maximum flash rate now being quoted, the results in tables 3a show that the cameras alone will not provide the answers.

Not much is known about the errors in the radar and the values quoted may well be optimistic. In the method of solution which estimates errors in position and velocity at the centre of a sampling interval, any systematic errors which can be expressed as a linear function of time are absorbed into these two quantities and only those of higher order, which are likely to be very small, will contribute to the acceleration error components. But in the second method where the burning period is considered as a whole and the integration is done from launch, systematic errors in ground instrumentation can be serious. Their contributions as shown in tables 3a and 3b were finally computed using only the residuals after the ballistic camera observations had been used to remove as far as possible the radar biases.

Errors in the estimation of drift, which are negligible for the z accelerometer calculations are as important as those from the ground instrumentation in the calculation of k_{0y}, k_{1y}, k_{2y}, and because the television cameras only record once a second, their contributions cannot be reduced by taking fixes more often than this since it will be necessary to use interpolated values.

8 CONCLUSION

If the accelerometers to be tested in the Black Knight inertia table trials are good to the accuracy specified i.e. 2×10^{-4}g bias and 2×10^{-4} scale error, with a possible non-linearity in the scale error, the method of assessing accelerometer accuracy by comparing the integrated output with position from ground instrumentation may prove to be unsatisfactory with the facilities at present available.

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ATTACHED:-

Appendices 1-5, Tables 1-5, Drgs. GW/P/9954 to 9956 Detachable Abstract Cards

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APPENDIX 1

ERRORS BETWEEN ACCELEROMETER OUTPUT AND TELEMETERED DIGITAL ACCELERATION

Source

(1)	Accelerometer voltage supply:	temperature variation vibration
(2)	Amplifier:	drift non-linearity temperature fluctuation } negligible vibration } negligible
(3)	A-D converter	
	Reference and bias voltages:	vibration temperature variation
	Resistors:	temperature variation (negligible) time variation
	Comparator:	rounding errors uncertainty in round off level time shift in round off level

Errors

Random	Description	σ
Comparator	round off	1.2×10^{-3} ft/sec^2
Comparator	uncertainty in drift of round off level	2.4×10^{-3} ft/sec^2
Highly correlated		
Amplifier	zero error	1×10^{-3} ft/sec^2
Bias voltage	temperature variation (1 cycle/min) vibration	1.1×10^{-3} ft/sec^2 $<1.6\times10^{-3}$ ft/sec^2
Resistors	variation between calibration and firing	For B.K. accelerations 3.1×10^{-3} ft/sec^2 t=0-90 secs 3.6×10^{-3} " t=90-120 secs 5.2×10^{-3} " t > 120 secs
Scale errors		
Amplifier	non-linearity	6.5×10^{-6}
Reference voltage	temperature variation (1 cycle/min) vibration	1.7×10^{-5} $< 1\times10^{-5}$
Accelerometer voltage	temperature variation vibration	1.7×10^{-5} $< 1\times10^{-5}$
Constant		
Comparator	uncertainty in round off level, < 1/10 quantum	2.4×10^{-3} ft/sec^2

Bias errors	σ
Random	2.7×10^{-3} ft/sec^2
Highly correlated	B.K. $\begin{cases} 3.5 \times 10^{-3} \text{ ft/sec}^2 & t = 0\text{–}90 \text{ secs} \\ 4 \times 10^{-3} \quad " & t = 90\text{–}120 \text{ secs} \\ 5.5 \times 10^{-3} \quad " & t > 120 \text{ secs} \end{cases}$
Constant	2.4×10^{-3} ft/sec^2
Scale errors	
Highly correlated	2.5×10^{-5}

NOTES: (1) If the errors in accelerometer voltage and amplifier are omitted on the grounds that these form an integral part of the accelerometer, the difference in σ for bias errors is negligible and for scale errors σ reduces to 2×10^{-5}.

(2) The errors from different sources have been combined on the assumption that they are independent. In the case of those caused by temperature variation some correlation is possible and this could cause a small variation in the σ for scale errors.

APPENDIX 2

ERRORS IN DRIFT ANGLES MEASURED BY TELEVISION CAMERAS

1 ERRORS IN CAMERA MEASUREMENTS

On the television screen the lines will be approximately two minutes of arc apart. If the focal length of the camera is f and

$$\frac{x_c}{f} = \alpha$$

$$\frac{z_c}{f} = \beta \qquad \text{(see Fig.3)}$$

as the star appears to move across the field of view of the camera the readings of β will be in steps and successive readings will be correlated, Figs.4 and 5.

The relation between the errors will depend on the drift rate, which is unknown, and the most that can be assumed is that the error in a single reading is random with a rectangular distribution and that errors in successive observations are correlated.

For α the errors may be assumed random and normally distributed with no correlation between successive values.

1.1 Relation between the direction cosines of the line of sight to a star referred to fixed space axes, and the measurements from a television camera

In the inertial table experiment the camera axes will be in the YZ plane inclined at 45° and 135° to OY. Fig.6.

Camera 1

The direction cosines of the camera axis referred to O,XYZ at launch are $(0, 1/\sqrt{2}, 1/\sqrt{2})$. Let (ℓ,m,n) be the direction cosines of the line of sight to a star referred to the fixed space axes and (ℓ_c,m_c,n_c) those referred to the camera axes.

Then

$$\ell_c = \ell$$

$$m_c = \frac{m+n}{\sqrt{2}}$$

$$n_c = \frac{-m+n}{\sqrt{2}} \quad .$$

Hence

$$\frac{\ell}{x_c} = \frac{m+n}{\sqrt{2}f} = \frac{-m+n}{\sqrt{2}\,z_c} = \text{constant } C$$

$$\ell = x_c\, c$$

$$m+n = \sqrt{2}\, f\, c$$

$$-m+n = \sqrt{2}\, z_c\, c$$

$$n = \frac{(f + z_c)}{\sqrt{2}}\, c$$

$$m = \frac{f-z_c}{\sqrt{2}}\, c \quad .$$

But $$\ell^2 + m^2 + n^2 = 1$$

$$\frac{1}{c^2} = x_c^2 + \frac{(f+z_c)^2}{2} + \frac{(f - z_c)^2}{2}$$

$$c = \frac{1}{\sqrt{f^2 + x_c^2 + z_c^2}}$$

and $$\ell = \frac{\frac{x_c}{f}}{\sqrt{1 + \left(\frac{x_c}{f}\right)^2 + \left(\frac{z_c}{f}\right)^2}}$$

$$m = \frac{1 - \frac{z_c}{f}}{\sqrt{2}\sqrt{1 + \left(\frac{x_c}{f}\right)^2 + \left(\frac{z_c}{f}\right)^2}}$$

$$n = \frac{1 + \frac{z_c}{f}}{\sqrt{2}\sqrt{1 + \left(\frac{x_c}{f}\right)^2 + \left(\frac{z_c}{f}\right)^2}} \quad .$$

The field of view of the camera is only $\pm 7.5^{\circ}$ and the movement within it is expected to be small. Hence, since the camera errors are quoted as angular errors, the direction cosines may justifiably be written

$$\ell_1 = \frac{\alpha_1}{\sqrt{1 + \alpha_1^2 + \beta_1^2}} = \alpha_1 \left(1 - \tfrac{1}{2}\left(\alpha_1^2 + \beta_1^2\right) + \ldots.\right)$$

$$m_1 = \frac{1 - \beta_1}{\sqrt{2}\sqrt{1 + \alpha_1^2 + \beta_1^2}} = \frac{(1 - \beta_1)(1 - \tfrac{1}{2}\left(\alpha_1^2 + \beta_1^2\right) + \ldots.}{\sqrt{2}}$$

$$n_1 = \frac{1 + \beta_1}{\sqrt{2}\sqrt{1 + \alpha_1^2 + \beta_1^2}} = \frac{(1 + \beta_1)(1 - \tfrac{1}{2}\left(\alpha_1^2 + \beta_1^2\right) + \ldots.}{\sqrt{2}}$$

where the suffix denotes camera 1

and

$$\delta\ell_1 \simeq \delta\alpha_1 \qquad \text{and} \qquad \sigma_{\ell_1}^2 = \sigma_{\alpha_1}^2$$

$$\delta m_1 \simeq -\frac{\delta\beta_1}{\sqrt{2}} \qquad \sigma_{m_1}^2 = \frac{\sigma_{\beta_1}^2}{2}$$

$$\delta n_1 \simeq +\frac{\delta\beta_1}{\sqrt{2}} \qquad \sigma_{n_1}^2 = \frac{\sigma_{\beta_1}^2}{2} .$$

In the same way it may be shown that for camera 2, the direction cosines of whose axis are $(0, -1/\sqrt{2}, 1/\sqrt{2})$, the direction cosines of a star are

$$\ell_2 = \frac{-\alpha_2}{\sqrt{1 + \alpha_2^2 + \beta_2^2}}$$

$$m_2 = \frac{-(1 - \beta_2)}{\sqrt{2}\sqrt{1 + \alpha_2^2 + \beta_2^2}}$$

$$n_2 = \frac{1 + \beta_2}{\sqrt{2}\sqrt{1 + \alpha_2^2 + \beta_2^2}}$$

and
$$\delta\ell_2 = -\delta\alpha_2 \qquad \sigma_{\ell_2}^2 = \sigma_{\alpha_2}^2$$

$$\delta m_2 = +\frac{\delta\beta_2}{\sqrt{2}} \qquad \sigma_{m_2}^2 = \frac{\sigma_{\beta_2}^2}{2}$$

$$\delta n_2 = +\frac{\delta\beta_2}{\sqrt{2}} \qquad \sigma_{n_2}^2 = \frac{\sigma_{\beta_2}^2}{2}$$

1.2 Calculation of drift angles in terms of $\ell_1 m_1 n_1$; $\ell_2 m_2 n_2$

If the directions of the camera axes remained constant relative to space axes a star in the field of view of the camera would appear stationary. If the camera axes drift the star will appear to move. Let ϕ_{xt}, ϕ_{yt}, ϕ_{zt} be the components of drift about the fixed space axes OXYZ. Then since ϕ is small, if we assume the camera axes to remain fixed and the star sight line to suffer rotations $-\phi_{xt}$, $-\phi_{yt}$, $-\phi_{zt}$, the apparent movement in the field of view of the camera will be the same. If $\ell'm'n'$ are the direction cosines of the new star sight line referred to the fixed space axes, ℓmn, $\ell'm'n'$ will be connected by the relation

$$\begin{bmatrix} +1 & -\phi_{zt} & +\phi_{yt} \\ +\phi_{zt} & +1 & -\phi_{xt} \\ -\phi_{yt} & +\phi_{xt} & +1 \end{bmatrix} \begin{bmatrix} \ell \\ m \\ n \end{bmatrix} = \begin{bmatrix} \ell' \\ m' \\ n' \end{bmatrix}$$

which may be written

$$\begin{bmatrix} 0 & +n & -m \\ -n & 0 & +\ell \\ +m & -\ell & 0 \end{bmatrix} \begin{bmatrix} \phi_{xt} \\ \phi_{yt} \\ \phi_{zt} \end{bmatrix} = \begin{bmatrix} \ell'-\ell \\ m'-m \\ n'-n \end{bmatrix} . \qquad \begin{matrix} (12) \\ (13) \\ (14) \end{matrix}$$

Because of the condition $\ell^2+m^2+n^2 = 1$ the three equations (12), (13), (14) are not independent. But taking (12) and (13) from the first camera and the corresponding ones from camera 2, there are four equations in three unknowns which may be solved by least squares to give estimates of ϕ_{xt}, ϕ_{yt}, ϕ_{zt}. Since the errors in $\ell_1 m_1 n_1$ do not have equal weights, it is better to revert to α and β and write the equations in the form

$$\begin{bmatrix} 0 & (1+\beta_1) & -(1-\beta_1) \\ -(1+\beta_1) & 0 & \sqrt{2}\alpha_1 \\ 0 & (1+\beta_2) & -(\beta_2-1) \\ -(1+\beta_2) & 0 & -\sqrt{2}\alpha_2 \end{bmatrix} \begin{bmatrix} \phi_{xt} \\ \phi_{yt} \\ \phi_{zt} \end{bmatrix} = \begin{bmatrix} \sqrt{\dfrac{1+\alpha_1^2+\beta_1^2}{1+\alpha_1'^2+\beta_1'^2}}\sqrt{2}\alpha_1' - \sqrt{2}\alpha_1 \\ \sqrt{\dfrac{1+\alpha_1^2+\beta_1^2}{1+\alpha_1'^2+\beta_1'^2}}(1-\beta_1') - (1-\beta_1) \\ \sqrt{\dfrac{1+\alpha_2^2+\beta_2^2}{1+\alpha_2'^2+\beta_2'^2}}(-\sqrt{2}\alpha_2') + \sqrt{2}\alpha_2 \\ \sqrt{\dfrac{1+\alpha_2^2+\beta_2^2}{1+\alpha_2'^2+\beta_2'^2}}(\beta_2'-1) - (\beta_2-1) \end{bmatrix} \tag{15}$$

To the first order

$$\sqrt{\frac{1+\alpha_1^2+\beta_1^2}{1+\alpha_1'^2+\beta_1'^2}} = 1$$

and the right hand side becomes

$$\begin{bmatrix} \sqrt{2}\alpha_1' - \sqrt{2}\alpha_1 \\ -\beta_1' + \beta_1 \\ -\sqrt{2}\alpha_2' + \sqrt{2}\,\alpha_2 \\ \beta_2' - \beta_2 \end{bmatrix}$$

which can be written

$$\begin{bmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1' - \alpha_1 \\ \beta_1' - \beta_1 \\ \alpha_2' - \alpha_2 \\ \beta_2' - \beta_2 \end{bmatrix}$$

and the equation (15) becomes

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & (1+\beta_1) & -(1-\beta_1) \\ -(1+\beta_1) & 0 & \sqrt{2}\alpha_1 \\ 0 & (1+\beta_2) & -(\beta_2-1) \\ -(1+\beta_2) & 0 & -\sqrt{2}\alpha_2 \end{bmatrix} \begin{bmatrix} \phi_{xt} \\ \phi_{yt} \\ \phi_{zt} \end{bmatrix} = \begin{bmatrix} \alpha_1' - \alpha_1 \\ \beta_1' - \beta_1 \\ \alpha_2' - \alpha_2 \\ \beta_2' - \beta_2 \end{bmatrix}$$

i.e.

$$\begin{bmatrix} 0 & \frac{1}{\sqrt{2}}(1+\beta_1) & -\frac{1}{\sqrt{2}}(1-\beta_1) \\ 1+\beta_1 & 0 & -\sqrt{2}\alpha_1 \\ 0 & -\frac{1}{\sqrt{2}}(1+\beta_2) & \frac{1}{\sqrt{2}}(\beta_2-1) \\ -(1+\beta_2) & 0 & -\sqrt{2}\alpha_2 \end{bmatrix} \begin{bmatrix} \phi_{xt} \\ \phi_{yt} \\ \phi_{zt} \end{bmatrix} = \begin{bmatrix} \alpha_1' - \alpha_1 \\ \beta_1' - \beta_1 \\ \alpha_2' - \alpha_2 \\ \beta_2' - \beta_2 \end{bmatrix}$$

where the α's and β's have the same variance. Expressing these equations in matrix form as

$$A_{4\times 3}\ \phi_3 = b_4$$

and solving by least squares, the normal equations (omitting the matrix dimensions) are given by

$$A^*A\ \phi = A^*b$$

and

$$\phi = (A^*A)^{-1}A^*b \quad .$$

Now for any non singular square matrix

$$\delta(c^{-1}) = -c^{-1}\ \delta c\ c^{-1}$$

so

$$\delta(A^*A)^{-1} = -(A^*A)^{-1}(A^*\delta A+\delta A^*A)(A^*A)^{-1}$$

and

$$\begin{aligned} \delta\phi &= \delta(A^*A)^{-1}A^*b + (A^*A)^{-1}\delta A^*b + (A^*A)^{-1}A^*\delta b \\ &= -(A^*A)^{-1}A^*\delta A(A^*A)^{-1}A^*b - (A^*A)^{-1}\delta A^*A(A^*A)^{-1}A^*b + (A^*A)^{-1}\delta A^*b + (A^*A)^{-1}A^*\delta b \quad . \end{aligned}$$

But $A_{4\times 3}(A^*A)^{-1}_{3\times 3}A^*_{3\times 4}$ is a unit matrix of dimensions 4.

Hence

$$\delta\phi = -(A^*A)^{-1} A^*\delta A(A^*A)^{-1}A^* b + (A^*A)^{-1} A^*\delta b$$

$$= -(A^*A)^{-1} A^*\delta A\ \phi + (A^*A)^{-1} A^*\delta b \quad .$$

But ϕ is known to be small, so to the first order

$$\delta\phi = (A^*A)^{-1} A^*\delta b ,$$

and the errors in ϕ depend upon the positions of the stars in the fields of view of the cameras, as well as on the reading errors.

Some idea of the variations in errors in drift angles due to star position may be found by considering images at the centre and boundaries of the field of view.

If suffix 1 denotes camera 1 and 2, camera 2, variances have been calculated for stars at O_1 with O_2, and A_1 with A_2, B_2, C_2, D_2 and O_2. (Fig.7).

The table below gives the weighting factors for σ^2 where σ^2 is the variance of the camera errors, assumed the same for α and β.

	O_1O_2	A_1A_2	A_1B_2	A_1C_2	A_1D_2	A_1O_2
$E(\delta\phi_x)^2$	1.0	0.78	0.98	1.05	0.85	0.90
$E(\delta\phi_y)^2$	2.0	1.56	1.97	1.97	1.56	1.77
$E(\delta\phi_z)^2$	2.0	2.43	1.94	1.96	2.65	2.27
$E(\delta\phi_x\delta\phi_y)$	0	0	0	0	0	0
$E(\delta\phi_x\delta\phi_z)$	0	0	0.04	0.36	0.43	0.21
$E(\delta\phi_y\delta\phi_z)$	0	0	0	0	0	0

APPENDIX 3

ERRORS IN POSITION FROM GROUND INSTRUMENTATION

1 BALLISTIC CAMERAS

Three Wild B.C.4 cameras with Aviotar lenses (f = 210 mm) lie in a ring of radius of the order of 30 n.m. about the launching area and four with Astrotar lenses (f = 310 mm) in an outer ring. The latter are much further away and less suitable to cover the B.K. burning period. With the following nomenclature

A	P.5	inner ring
B	Woomera Tech. Area	
C	Coondambo	
D	Maralinga	outer ring
E	Oodnadattor	
F	Bereford	
G	Ceduna	

and assuming an r.m.s. error of one second of arc at each camera, the following errors in position result from weighted least squares solutions in three dimensions for a standard Black Knight trajectory.

Z	13,000 ft			53,000 ft			339,000 ft		
Stations	σ_x	σ_y	σ_z	σ_x	σ_y	σ_z	σ_x	σ_y	σ_z
A,B,C	0.7	0.9	0.5	0.6	0.9	0.5	1.2	1.2	2.5
A,B	0.9	1.9	0.6	0.8	1.4	0.6	1.4	1.5	3.2
D,E,F,G	3.0	4.3	2.3	3.0	4.3	2.3	3.0	4.0	3.0

Standard deviations in feet per sec of arc.

The accuracy generally quoted for the cameras with Aviator lenses is 5 seconds of arc random error. Very little experience has been gained by G.W. Dept., R.A.E. in the use of the cameras on trials. One camera plate which has been calibrated here showed an r.m.s. error over all star points of 5 microns ($\simeq$ 5 seconds of arc) when over 100 stars were used to compute the plate constants. If however as few as 12 stars were used to calibrate it was possible to introduce a bias of the order of 3 seconds which would vary over the plate.

One set of observations[3] has so far been received from W.R.E. These were angles from the three inner ring cameras used for the B.K.05 firing. The results showed a random error of approximately 5 seconds of arc and in addition a bias of the same order, which was not constant.

The effect of errors arising in the survey of the camera sites has been investigated and found to be small compared with camera errors.

The sampling rate of the ballistic cameras is limited by the frequency of the electronic flash unit. This is quoted as one every 5 seconds.

2 F.P.S. 16 RADAR

All information on the accuracy of the F.P.S.16 radar has been taken from the R.C.A. report on Evaluation and Analysis of Radar Performance[3]. The errors appear to fall into three groups.

(1) Large errors due to propagation and dynamic lag for which it should be possible to apply corrections.

(2) Bias, both in range and angle which will vary from trial to trial but should remain constant in any one firing. The effect of this on a rectangular co-ordinate is approximately linear with time for Black Knight during a sampling period and as such will contribute no error to k.

(3) "Noise". The noise is not truly random but has low frequency components. This reduces the value of using a high sampling rate.

The figures deduced for bias are of the order of 0.07 mil in elevation and 8.6 ft in range and for noise 0.04 mils in angle and 8.5 ft in range. These have been reached by a combination of theory and the results of calibration trials on relatively short range targets, with the longer ranges generally associated with low elevations.

The maximum sampling rate is 60/sec and that proposed by W.R.E., 40/sec.

APPENDIX 4

THE USE OF BALLISTIC CAMERA OBSERVATIONS TO REDUCE THE RADAR BIAS ERRORS

1 ESTIMATION OF BIAS

1.1 The siting of the B.C.4 cameras and the F.P.S.16 radar at Woomera is shown in Fig.1. During the thrust period of Black Knight best coverage is given by the three cameras in the inner ring and in estimating positional errors from optical data it has been assumed that observations are available from all three stations. These have been used to compute radar co-ordinates of the missile, R, θ, ϕ where θ is the elevation and ϕ azimuth measured from the y axis, and their covariance matrices for various positions along the trajectory, with the assumption that the cameras have a random angular error of 5 seconds and no bias.

At any time t let (R, θ, ϕ) Fig.2 be the true radar co-ordinates, $(R_r \theta_r \phi_r)$ those given by the radar and $(R_c \theta_c \phi_c)$ the values calculated from camera data. Let ΔR be the systematic error in radar range and ΔR_{rt} the random error and let ΔR_{ct} be the corresponding random error in camera range at time t. Then

$$R_{rt} = R + \Delta R + \Delta R_{rt}$$

and

$$R_{ct} = R + \Delta R_{ct}$$

Therefore

$$\Delta R = R_{rt} - R_{ct} + \varepsilon_t$$

where ε_t is random if we assume no bias in the camera observations. Taking a number of times t and solving the resultant equations by least squares, the best value for ΔR is the mean

i.e.

$$\Delta R = \frac{\sum_{i=1}^{n} (R_{rt_1} - R_{ct_1})}{n}$$

and similarly for $\Delta\theta$ and $\Delta\phi$.

1.2 Residual bias errors

The random errors in radar range are virtually independent of distance and so have a constant variance σ_{Rr}^2.

Hence

$$\sigma_{\Delta R}^2 = \frac{1}{n^2}\left[n\,\sigma_{Rr}^2 + \sum_{i=1}^{n} \sigma_{Rc_i}^2\right]$$

and

$$\sigma_{\Delta\theta}^2 = \frac{1}{n^2}\left[n\,\sigma_{\theta r}^2 + \sum_{i=1}^{n} \sigma_{\theta c_i}^2\right]$$

$$\sigma_{\Delta\phi}^2 = \frac{1}{n^2}\left[n\,\sigma_{\phi r}^2 + \sum_{i=1}^{n} \sigma_{\phi c_i}^2\right] .$$

The errors in ΔR, $\Delta\theta$ and $\Delta\phi$ are now correlated. For if $\delta(\Delta R)$, $\delta(\Delta\theta)$ are errors in ΔR, $\Delta\theta$ due to random errors in R_{rt}, R_{ct}, θ_{rt}, θ_{ct},

$$\delta(\Delta R)\ \delta(\Delta\theta) = \frac{1}{n^2}\left[\sum_i \delta R_{rt_1} - \sum_i \delta R_{ct_i}\right]\left[\sum_i \theta_{rt_1} - \sum_i \delta\theta_{ct_1}\right]$$

Here the errors in radar are independent of camera errors and the radar range errors are assumed to be independent of radar angular errors and so

$$\left(\sigma_{\Delta R}\ \sigma_{\Delta\theta}\right) = \frac{1}{n^2}\sum_i \left(\sigma_{Rc_i}\ \sigma_{\theta c_i}\right) .$$

Likewise

$$\left(\sigma_{\Delta R}\ \sigma_{\Delta\phi}\right) = \frac{1}{n^2}\sum_i \left(\sigma_{Rc_i}\ \sigma_{\phi c_i}\right)$$

$$\left(\sigma_{\Delta\theta}\ \sigma_{\Delta\phi}\right) = \frac{1}{n^2}\sum_i \left(\sigma_{\theta c_i}\ \sigma_{\phi c_i}\right) .$$

2 RESIDUAL SYSTEMATIC ERRORS IN y AND z

From Fig.2

$$\Delta y = \Delta R \cos\theta \cos\phi - R \sin\theta \cos\phi\ \Delta\theta - R \cos\theta \sin\phi\ \Delta\phi$$

$$\Delta z = \Delta R \sin\theta + R \cos\theta\ \Delta\theta .$$

Although the systematic errors in the radar vary from day to day, on any one trial they may be considered to be constant. Hence if $\sigma^2_{\Delta y}$, $\sigma^2_{\Delta z}$ are the variances of the errors in y and z caused by residual systematic radar errors after correction by ballistic cameras

$$\sigma^2_{\Delta y} = \sigma^2_{\Delta R}\cos^2\theta\cos^2\phi + \sigma^2_{\Delta\theta}R^2\sin^2\theta\cos^2\phi + \sigma^2_{\Delta\phi}R^2\cos^2\theta\sin^2\phi$$

$$- 2(\sigma_{\Delta R}\,\sigma_{\Delta\theta})R\sin\theta\cos\theta\cos^2\phi - 2(\sigma_{\Delta R}\sigma_{\Delta\phi})R\cos^2\theta\cos\phi\sin\phi$$

$$+ 2(\sigma_{\Delta\theta}\sigma_{\Delta\phi})R^2\sin\theta\cos\theta\sin\phi\cos\phi$$

$$\sigma^2_{\Delta z} = \sigma^2_{\Delta R}\sin^2\theta + \sigma^2_{\Delta\theta}R^2\cos^2\theta + (\sigma_{\Delta R}\sigma_{\Delta\theta})R\sin 2\theta \quad .$$

Using the covariance matrices for $(R_{ct}\ \theta_{ct}\ \phi_{ct})$ given in tables 5,

$$\sigma^2_{\Delta R} = 4.1\ \text{ft}^2$$

$$\sigma^2_{\Delta\theta} = 0.9 \times 10^{-10}\ \text{radians}^2$$

$$\sigma^2_{\Delta\phi} = 1.3 \times 10^{-10}\ \text{radians}^2$$

$$\sigma_{\Delta R}\sigma_{\Delta\theta} = 1.7 \times 10^{-6}\ \text{ft radians}$$

$$\sigma_{\Delta R}\sigma_{\Delta\phi} = 1.0 \times 10^{-6}\ \text{ft radians}$$

$$\sigma_{\Delta\theta\ \Delta\phi} = -0.73 \times 10^{-10}\ \text{radians}^2$$

These give computed values of σ^2_{zt} increasing from 1.5 at t = 35 secs to 4.6 ft^2 at t = 135 secs and σ^2_{yt} remaining practically constant at 2.1 ft^2.

APPENDIX 5

ERRORS CAUSED BY k_2 WHEN THERE ARE OSCILLATIONS IN THE ACCELERATION OF FREQUENCY HIGH COMPARED WITH THE TELEMETRY SAMPLING RATE

Let

$$\ddot{x} = f(t) + \sum a_i \cos(\omega_i t + \phi_i) .$$

Error due to $k_2 = k_2 \ddot{x}^2$

$$= k_2 \left[f(t) + \sum a_i \cos(\omega_i t + \phi_i) \right]^2$$

$$= k_2 \left[(f(t))^2 + 2f(t) \sum a_i \cos(\omega_i t+\phi_i) + \sum a_i^2 \cos^2(\omega_i t+\phi_i) \right.$$

$$\left. + \sum_i \sum_j a_i a_j \cos(\omega_i t+\phi_i)\cos(\omega_j t+\phi_j) \right]$$

Error due to $k_1 = k_1 \ddot{x}$

$$= k_1 \left[f(t) + \sum a_i \cos(\omega_i t+\phi_i) \right]$$

Error due to $k_0 = k_0$.

Let $\ddot{x}_r$ be the recorded acceleration, since this is the mean value taken over $\frac{1}{15}$ sec.,

/Equation

$$\ddot{x}_r = \frac{\int_t^{t+\frac{1}{15}} (\ddot{x}+\varepsilon)\,dt}{\frac{1}{15}}$$

where ε is the contribution from acc. errors,

$$= 15\Bigg[\int_t^{t+\frac{1}{15}} f(t)dt + \int_t^{t+\frac{1}{15}} \sum_i a_i \cos(\omega_i t+\phi_i)dt + \frac{k_0}{15} + k_1 \int_t^{t+\frac{1}{15}} f(t)dt$$

$$+k_1 \int_t^{t+\frac{1}{15}} \sum a_i \cos(\omega_i t+\phi_i)dt + k_2 \int_t^{t+\frac{1}{15}} [f(t)]^2 dt + 2k_2 \int_t^{t+\frac{1}{15}} f(t) \sum a_i \cos(\omega_i t+\phi_i)\,dt$$

$$+k_2 \int \sum a_i^2 \cos^2(\omega_i t+\phi_i)dt + k_2 \int \sum_i \sum_j a_i a_j \cos(\omega_i t+\phi_i)\cos(\omega_j t+\phi_j)\,dt\Bigg] .$$

Now if we assume that ω_i is large compared with the telemetry sampling rate

$$\frac{\int_t^{t+\frac{1}{15}} \sum_i a_i \cos(\omega_i t+\phi_i)dt}{\frac{1}{15}} = \frac{\sum_i \int_t^{t+\frac{1}{15}} a_i \cos(\omega_i t+\phi_i)dt}{\frac{1}{15}} \simeq 0$$

and since ϕ_i, ϕ_j are random

$$\sum_i \sum_j a_i a_j \cos(\omega_i t+\phi_i) \cos(\omega_j t+\phi_j) \rightarrow 0$$

Hence

$$\ddot{x}_r = \overline{f(t)} + k_0 + k_1\,\overline{f(t)} + k_2\,\overline{[f(t)]^2} + k_2 \sum_i \frac{a_i^2}{2}\,.$$

But f(t) changes slowly, so in 1/15 sec

$$\overline{[f(t)]^2} \simeq \left(\overline{f(t)}\right)^2$$

and we may write

$$\ddot{x}_r = \overline{f(t)} + k_0 + k_1\,\overline{f(t)} + k_2\left(\overline{f(t)}\right)^2 + k_2 \sum_i \frac{a_i^2}{2}\,.$$

When there is no error in the accelerometer, i.e. $k_0 = k_1 = k_2 = 0$, $\bar{\ddot{x}} = \overline{f(t)}$, and the k's have been computed on the assumption that

$$\ddot{x}_r = \overline{f(t)} + k_0 + k_1\,\overline{f(t)} + k_2\left(\overline{f(t)}\right)^2 ,$$

ignoring the term $k_2 \sum_i \frac{a_i^2}{2}$. If $\frac{\sum a_i^2}{2}$ remains constant throughout the burning period $k_2 \frac{\sum_i a_i^2}{2}$ will appear as an error in k_0, since they will not be separable. If, on the other hand, the power of the spectrum changes with time, it will be necessary to use weights and errors will appear in the estimates of k_1 and k_2 as well as in k_0.

<u>Estimation of $\frac{\sum a_i^2}{2}$</u>

A vibration record relevant to the longitudinal accelerometer has been analysed using SPADA for B.K.03. The original record consisted of seven two second bursts, giving vibration frequencies of 20 c/s upwards but the analysis is unreliable below 100 c/s. The mean square value of the output remained

relatively constant except for one burst at 50 sec after launch when it increased by a factor of three. Ignoring this, and taking an average value of spectral density from the curves as approximately $8 \times 10^{-4} g^2/c/s$ and integrating from 20-400 cycles/sec the maximum frequency to which the Kearfott accelerometers are sensitive, $\frac{\sum a_i^2}{2} \simeq 3 \times 10^{-1} g^2$. The estimated value of k_2 is 2×10^{-7} and thus the error in k_0 due to a combination of k_2 and oscillations in acceleration would be approximately

$$3 \times 10^{-1} \times 2 \times 10^{-7} \times 1.024 \times 10^{3} \simeq 6 \times 10^{-5} \text{ ft/sec}^2$$

and this is negligible.

TABLE 1a

Components of error in k_z

Period 10 secs

τ secs	T secs	10 weight	10 σ^2	30 weight	30 σ^2	50 weight	50 σ^2	70 weight	70 σ^2	90 weight	90 σ^2	110 weight	110 σ^2	130 weight	130 σ^2
1.0	B_1	7.23×10^{-5}	7×10^{-6}	5.4×10^{-5}	7×10^{-6}	4.4×10^{-5}	7×10^{-6}	4.5×10^{-5}	7×10^{-6}	2.0×10^{-5}	7×10^{-6}	7.7×10^{-6}	7×10^{-6}	3.0×10^{-6}	7×10^{-6}
	B_2	5.37×10^{-4}	1.8×10^{-5}	4.0×10^{-4}	1.8×10^{-5}	3.3×10^{-4}	1.8×10^{-5}	3.4×10^{-4}	1.8×10^{-5}	1.5×10^{-4}	2.2×10^{-5}	5.8×10^{-5}	3.6×10^{-5}	2.2×10^{-5}	3.6×10^{-5}
	C_1	1.34×10^{-1}	-	1.3×10^{-1}		1.3×10^{-1}		1.4×10^{-1}		1.4×10^{-1}		1.4×10^{-1}		1.4×10^{-1}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	2.5×10^{-6}	25	1.8×10^{-6}	25	1.5×10^{-6}	25	1.6×10^{-6}	30	6.9×10^{-7}	38	2.7×10^{-7}	59	1.5×10^{-7}	72
	D_2	0		0		0		0		0		0		0	
	E_1	1.06×10^{-5}	1.7×10^{-7}	2.1×10^{-6}	1.7×10^{-7}	2.9×10^{-5}	1.7×10^{-7}	2.8×10^{-5}	1.7×10^{-7}	6.2×10^{-5}	1.7×10^{-7}	9.5×10^{-5}	1.7×10^{-7}	1.2×10^{-4}	1.7×10^{-7}
	E_2	7.85×10^{-5}		1.5×10^{-4}		2.1×10^{-4}		2.1×10^{-4}		4.6×10^{-4}		7.1×10^{-4}		8.9×10^{-4}	
	F_2	7.85×10^{-5}	5.8×10^{-10}	1.5×10^{-4}	5.8×10^{-10}	2.1×10^{-4}	5.8×10^{-10}	2.1×10^{-4}	5.8×10^{-10}	4.6×10^{-4}	5.8×10^{-10}	7.1×10^{-4}	5.8×10^{-10}	8.9×10^{-4}	5.8×10^{-10}
	RMS error in k_z	7.91×10^{-3}		6.71×10^{-3}		6.13×10^{-3}		6.93×10^{-3}		5.12×10^{-3}		3.99×10^{-3}		3.29×10^{-3}	
0.5	B_1	3.7×10^{-5}	7×10^{-6}	2.8×10^{-5}	7×10^{-6}	2.3×10^{-5}	7×10^{-6}	2.3×10^{-5}	7×10^{-6}	1.0×10^{-5}	7×10^{-6}	3.9×10^{-6}	7×10^{-6}	1.5×10^{-6}	7×10^{-6}
	B_2	5.4×10^{-4}	1.8×10^{-5}	4.0×10^{-4}	1.8×10^{-5}	3.3×10^{-4}	1.8×10^{-5}	3.4×10^{-4}	1.8×10^{-5}	1.5×10^{-4}	2.2×10^{-5}	5.8×10^{-5}	3.6×10^{-5}	2.2×10^{-5}	3.6×10^{-5}
	C_1	6.9×10^{-2}		6.9×10^{-2}		6.9×10^{-2}		6.9×10^{-2}		6.9×10^{-2}		6.9×10^{-2}		6.9×10^{-2}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	1.5×10^{-6}	25	1.2×10^{-6}	25	9.4×10^{-7}	25	9.6×10^{-7}	30	4.2×10^{-7}	38	1.6×10^{-7}	59	6.4×10^{-8}	72
	D_2	0		0		0		0		0		0		0	
	E_1	1.1×10^{-5}	1.7×10^{-7}	2.1×10^{-5}	1.7×10^{-7}	2.9×10^{-5}	1.7×10^{-7}	2.8×10^{-5}	1.7×10^{-7}	6.2×10^{-5}	1.7×10^{-7}	9.6×10^{-5}	1.7×10^{-7}	1.2×10^{-4}	1.7×10^{-7}
	E_2	7.8×10^{-5}		1.5×10^{-4}		2.1×10^{-4}		2.1×10^{-4}		4.5×10^{-4}		7.0×10^{-4}		8.8×10^{-4}	
	F_2	7.8×10^{-5}	5.8×10^{-10}	1.5×10^{-4}	5.8×10^{-10}	2.1×10^{-4}	5.8×10^{-10}	2.1×10^{-4}	5.8×10^{-10}	4.5×10^{-4}	5.8×10^{-10}	7.0×10^{-4}	5.8×10^{-10}	8.8×10^{-4}	5.8×10^{-10}
	RMS error in k_z	6.12×10^{-3}		5.48×10^{-3}		4.85×10^{-3}		5.37×10^{-3}		4.0×10^{-3}		3.07×10^{-3}		2.15×10^{-3}	

B_1 - Random, B_2 - Constant: zero error in A-D converter.
C_1 - Random, C_2 - Constant: scale error in A-D convertor.
D_1 - Random, D_2 - Constant: ground instrument error.
E_1 - Random, E_2 - Constant: television camera error.

F_2 - measurement of misalignment error.

T = seconds of sampling period.

τ = sampling interval.

TABLE 1a (Cont'd)

Period 20 secs

τ secs \ T secs		10		30		50		70		90		110		130	
		weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2
1.0	B_1	3.69×10^{-5}	7×10^{-6}	2.77×10^{-5}	7×10^{-6}	2.26×10^{-5}	7×10^{-6}	2.31×10^{-5}	7×10^{-6}	1.02×10^{-5}	7×10^{-6}	3.94×10^{-6}	7×10^{-6}	1.52×10^{-6}	7×10^{-6}
	B_2	5.37×10^{-4}	1.8×10^{-5}	4.03×10^{-4}	1.8×10^{-5}	3.29×10^{-4}	1.8×10^{-5}	3.36×10^{-4}	1.8×10^{-5}	1.48×10^{-4}	2.2×10^{-5}	5.73×10^{-5}	3.6×10^{-5}	2.23×10^{-5}	3.6×10^{-5}
	C_1	6.87×10^{-2}		6.87×10^{-2}		6.87×10^{-2}		6.87×10^{-2}		6.91×10^{-2}		6.9×10^{-2}		6.93×10^{-2}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	9.6×10^{-8}	25	7.2×10^{-8}	25	5.8×10^{-8}	25	6.0×10^{-8}	30	2.6×10^{-8}	38	1.0×10^{-8}	59	4.0×10^{-9}	72
	D_2	0		0		0		0		0		0		0	
	E_1	5.43×10^{-6}	1.7×10^{-7}	1.06×10^{-5}	1.7×10^{-7}	1.46×10^{-5}	1.7×10^{-7}	1.44×10^{-5}	1.7×10^{-7}	3.20×10^{-5}	1.7×10^{-7}	4.89×10^{-5}	1.7×10^{-7}	6.16×10^{-5}	1.7×10^{-7}
	E_2	7.85×10^{-5}		1.53×10^{-4}		2.13×10^{-4}		2.08×10^{-4}		4.58×10^{-4}		7.05×10^{-4}		8.86×10^{-4}	
	F_2	7.85×10^{-5}	5.8×10^{-10}	1.53×10^{-4}	5.8×10^{-10}	2.13×10^{-4}	5.8×10^{-10}	2.08×10^{-4}	5.8×10^{-10}	4.58×10^{-4}	5.8×10^{-10}	7.05×10^{-4}	5.8×10^{-10}	8.86×10^{-4}	5.8×10^{-10}
	RMS error in k_z	1.55×10^{-3}		1.34×10^{-3}		1.2×10^{-3}		1.3×10^{-3}		9.96×10^{-4}		7.7×10^{-4}		5.4×10^{-4}	
0.5	B_1	1.9×10^{-5}	7×10^{-6}	1.4×10^{-5}	7×10^{-6}	1.2×10^{-5}	7×10^{-6}	1.2×10^{-5}	7×10^{-6}	5.2×10^{-6}	7×10^{-6}	2.0×10^{-6}	7×10^{-6}	7.8×10^{-7}	7×10^{-6}
	B_2	5.4×10^{-4}	1.8×10^{-5}	4.0×10^{-4}	1.8×10^{-5}	3.3×10^{-4}	1.8×10^{-5}	3.4×10^{-4}	1.8×10^{-5}	1.5×10^{-4}	2.2×10^{-5}	5.7×10^{-5}	3.6×10^{-5}	2.2×10^{-5}	3.6×10^{-5}
	C_1	3.5×10^{-2}		3.5×10^{-2}		3.5×10^{-2}		3.5×10^{-2}		3.5×10^{-2}		3.5×10^{-2}		3.5×10^{-2}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	5.4×10^{-8}	25	4×10^{-8}	25	3.3×10^{-8}	25	3.4×10^{-8}	30	1.5×10^{-8}	38	6.0×10^{-9}	59	2.0×10^{-9}	72
	D_2	0		0		0		0		0		0		0	
	E_1	5.5×10^{-6}	1.7×10^{-7}	1.1×10^{-5}	1.7×10^{-7}	1.5×10^{-5}	1.7×10^{-7}	1.5×10^{-5}	1.7×10^{-7}	3.2×10^{-5}	1.7×10^{-7}	5.0×10^{-5}	1.7×10^{-7}	6.3×10^{-5}	1.7×10^{-7}
	E_2	7.8×10^{-5}		1.5×10^{-4}		2.1×10^{-4}		2.1×10^{-4}		4.6×10^{-4}		7.0×10^{-4}		8.9×10^{-4}	
	F_2	7.8×10^{-5}	5.8×10^{-10}	1.5×10^{-4}	5.8×10^{-10}	2.1×10^{-4}	5.8×10^{-10}	2.1×10^{-4}	5.8×10^{-10}	4.6×10^{-4}	5.8×10^{-10}	7.0×10^{-4}	5.8×10^{-10}	8.9×10^{-4}	5.8×10^{-10}
	RMS error in k_z	1.17×10^{-3}		1.00×10^{-3}		9.12×10^{-4}		1.01×10^{-3}		7.58×10^{-4}		5.97×10^{-4}		3.81×10^{-4}	

B_1 - Random, B_2 - Constant: zero error in A-D converter.
C_1 - Random, C_2 - Constant: scale error in A-D. converter.
D_1 - Random, D_2 - Constant: ground instrument error.
E_1 - Random, E_2 - Constant: Television camera error.

F_2 - Acc. misalignment error.

T = seconds of sampling period.

τ = sampling interval.

TABLE 1a (Cont'd)

Period 30 secs

τ secs	T secs	10 weight	10 σ^2	30 weight	30 σ^2	50 weight	50 σ^2	70 weight	70 σ^2	90 weight	90 σ^2	110 weight	110 σ^2	130 weight	130 σ^2
1.0	B_1	2.5×10^{-5}	7×10^{-6}	1.9×10^{-5}	7×10^{-6}	1.5×10^{-5}	7×10^{-6}	1.6×10^{-5}	7×10^{-6}	6.8×10^{-6}	7×10^{-6}	2.7×10^{-6}	7×10^{-6}	1.0×10^{-6}	7×10^{-6}
	B_2	5.4×10^{-4}	1.8×10^{-5}	4.0×10^{-4}	1.8×10^{-5}	3.3×10^{-4}	1.8×10^{-5}	3.4×10^{-4}	1.8×10^{-5}	1.5×10^{-4}	2.2×10^{-5}	5.7×10^{-5}	3.6×10^{-5}	2.2×10^{-5}	3.6×10^{-5}
	C_1	4.6×10^{-2}		4.6×10^{-2}		4.6×10^{-2}		4.6×10^{-2}		4.7×10^{-2}		4.7×10^{-2}		4.7×10^{-2}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	1.4×10^{-8}	25	1.0×10^{-8}	25	8.0×10^{-9}	25	9.0×10^{-9}	30	4×10^{-9}	38	1.0×10^{-9}	59	6.6×10^{-10}	72
	D_2	0		0		0		0		0		0		0	
	E_1	3.7×10^{-6}	1.7×10^{-7}	7.2×10^{-6}	1.7×10^{-7}	9.8×10^{-6}	1.7×10^{-7}	9.8×10^{-6}	1.7×10^{-7}	2.2×10^{-5}	1.7×10^{-7}	3.3×10^{-5}	1.7×10^{-7}	4.2×10^{-5}	1.7×10^{-7}
	E_2	7.9×10^{-5}		1.5×10^{-4}		2.1×10^{-4}		2.1×10^{-4}		4.6×10^{-4}		7.1×10^{-4}		8.9×10^{-4}	
	F_2	7.9×10^{-5}	5.8×10^{-10}	1.5×10^{-4}	5.8×10^{-10}	2.1×10^{-4}	5.8×10^{-10}	2.1×10^{-4}	5.8×10^{-10}	4.6×10^{-4}	5.8×10^{-10}	7.1×10^{-4}	5.8×10^{-10}	8.9×10^{-4}	5.8×10^{-10}
	RMS error in k_z	6.00×10^{-4}		5.08×10^{-4}		4.55×10^{-4}		5.26×10^{-4}		3.95×10^{-4}		2.48×10^{-4}		2.21×10^{-4}	
0.5	B_1	1.3×10^{-5}	7×10^{-6}	9.4×10^{-6}	7×10^{-6}	7.7×10^{-6}	7×10^{-6}	7.9×10^{-6}	7×10^{-6}	3.5×10^{-6}	7×10^{-6}	1.3×10^{-6}	7×10^{-6}	5.2×10^{-7}	7×10^{-6}
	B_2	5.4×10^{-4}	1.8×10^{-5}	4.0×10^{-4}	1.8×10^{-5}	3.3×10^{-4}	1.8×10^{-5}	3.4×10^{-4}	1.8×10^{-5}	1.5×10^{-4}	2.2×10^{-5}	5.7×10^{-5}	3.6×10^{-5}	2.2×10^{-5}	3.6×10^{-5}
	C_1	2.4×10^{-2}		2.4×10^{-2}		2.4×10^{-2}		2.4×10^{-2}		2.4×10^{-2}		2.4×10^{-2}		2.4×10^{-2}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	7.0×10^{-9}	25	6.0×10^{-9}	25	4.0×10^{-9}	25	5.0×10^{-9}	30	2.0×10^{-9}	38	8.0×10^{-10}	59	3.0×10^{-10}	72
	D_2	0		0		0		0		0		0		0	
	E_1	3.7×10^{-6}	1.7×10^{-7}	7.3×10^{-6}	1.7×10^{-7}	9.9×10^{-6}	1.7×10^{-7}	9.9×10^{-6}	1.7×10^{-7}	2.2×10^{-5}	1.7×10^{-7}	3.4×10^{-5}	1.7×10^{-7}	4.3×10^{-5}	1.7×10^{-7}
	E_2	7.9×10^{-5}		1.5×10^{-4}		2.1×10^{-4}		2.1×10^{-4}		4.6×10^{-4}		7.1×10^{-4}		8.9×10^{-4}	
	F_2	7.9×10^{-5}	5.8×10^{-10}	1.5×10^{-4}	5.8×10^{-10}	2.1×10^{-4}	5.8×10^{-10}	2.1×10^{-4}	5.8×10^{-10}	4.6×10^{-4}	5.8×10^{-10}	7.1×10^{-4}	5.8×10^{-10}	8.9×10^{-4}	5.8×10^{-10}
	RMS error in k_z	4.31×10^{-4}		3.97×10^{-4}		3.27×10^{-4}		3.96×10^{-4}		2.83×10^{-4}		2.2×10^{-4}		1.5×10^{-4}	

B_1 - Random, B_2 - Constant: zero error in A-D converter.
C_1 - Random, C_2 - Constant: scale error in A-D convertor.
D_1 - Random, D_2 - Constant: ground instrument errors.
E_1 - Random, E_2 - Constant: television camera error.

F_2 - acc. misalignment error.

T = seconds of sampling period.

τ = sampling interval.

TABLE 1a (Cont'd)

Period 40 secs

τ secs	T secs	10 weight	10 σ^2	30 weight	30 σ^2	50 weight	50 σ^2	70 weight	70 σ^2	90 weight	90 σ^2	110 weight	110 σ^2	130 weight	130 σ^2
1.0	B_1	1.9×10^{-5}	7×10^{-6}	1.4×10^{-5}	7×10^{-6}	1.2×10^{-5}	7×10^{-6}	1.2×10^{-5}	7×10^{-6}	5.1×10^{-6}	7×10^{-6}	2.0×10^{-6}	7×10^{-6}	2.2×10^{-5}	7×10^{-6}
	B_2	5.4×10^{-4}	1.8×10^{-5}	4.0×10^{-4}	1.8×10^{-5}	3.3×10^{-4}	1.8×10^{-5}	3.4×10^{-4}	1.8×10^{-5}	1.5×10^{-4}	2.2×10^{-5}	5.7×10^{-5}	3.6×10^{-5}	7.7×10^{-7}	3.6×10^{-5}
	C_1	3.5×10^{-2}		3.5×10^{-2}		3.5×10^{-2}		3.5×10^{-2}		3.6×10^{-2}		3.6×10^{-2}		3.6×10^{-2}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	3.4×10^{-9}	25	2.5×10^{-9}	25	2.1×10^{-9}	25	2.1×10^{-9}	30	9.2×10^{-10}	38	3.6×10^{-10}	59	1.4×10^{-10}	72
	D_2	0		0		0		0		0		0		0	
	E_1	2.8×10^{-6}	1.7×10^{-7}	5.5×10^{-6}	1.7×10^{-7}	7.4×10^{-6}	1.7×10^{-7}	7.5×10^{-6}	1.7×10^{-7}	1.7×10^{-5}	1.7×10^{-7}	7.1×10^{-4}	1.7×10^{-7}	3.3×10^{-5}	1.7×10^{-7}
	E_2	7.9×10^{-5}		1.5×10^{-4}		2.1×10^{-4}		2.1×10^{-4}		4.6×10^{-4}		2.6×10^{-5}		8.9×10^{-4}	
	F_2	7.9×10^{-5}	5.8×10^{-10}	1.5×10^{-4}	5.8×10^{-10}	2.1×10^{-4}	5.8×10^{-10}	2.1×10^{-4}	5.8×10^{-10}	4.6×10^{-4}	5.8×10^{-10}	2.6×10^{-5}	5.8×10^{-10}	8.9×10^{-4}	5.8×10^{-10}
	RMS error in k_z	3.09×10^{-4}		2.65×10^{-4}		2.43×10^{-4}		2.64×10^{-4}		1.97×10^{-4}		1.6×10^{-4}		1.04×10^{-4}	
0.5	B_1	9.5×10^{-6}	7×10^{-6}	7.1×10^{-6}	7×10^{-6}	5.8×10^{-6}	7×10^{-6}	5.9×10^{-6}	7×10^{-6}	2.6×10^{-6}	7×10^{-6}	1.0×10^{-6}	7×10^{-6}	3.9×10^{-7}	7×10^{-6}
	B_2	5.4×10^{-4}	1.8×10^{-5}	4.0×10^{-4}	1.8×10^{-5}	3.3×10^{-4}	1.8×10^{-5}	3.4×10^{-4}	1.8×10^{-5}	1.5×10^{-4}	2.2×10^{-5}	5.7×10^{-5}	3.6×10^{-5}	2.2×10^{-5}	3.6×10^{-5}
	C_1	1.8×10^{-2}		1.8×10^{-2}		1.8×10^{-2}		1.8×10^{-2}		1.8×10^{-2}		1.8×10^{-2}		1.8×10^{-2}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	1.8×10^{-9}	25	1.3×10^{-9}	25	1.1×10^{-9}	25	1.1×10^{-9}	30	4.9×10^{-10}	38	1.9×10^{-10}	59	7.3×10^{-11}	72
	D_2	0		0		0		0		0		0		0	
	E_1	1.4×10^{-6}	1.7×10^{-7}	2.8×10^{-6}	1.7×10^{-6}	3.7×10^{-6}	1.7×10^{-7}	3.8×10^{-6}	1.7×10^{-7}	8.6×10^{-5}	1.7×10^{-7}	3.6×10^{-4}	1.7×10^{-7}	1.6×10^{-5}	1.7×10^{-7}
	E_2	7.9×10^{-5}		1.5×10^{-4}		2.1×10^{-4}		2.1×10^{-4}		4.6×10^{-4}		2.6×10^{-5}		8.9×10^{-4}	
	F_2	7.9×10^{-5}	5.8×10^{-10}	1.5×10^{-4}	5.8×10^{-10}	2.1×10^{-4}	5.8×10^{-10}	2.1×10^{-4}	5.8×10^{-10}	4.6×10^{-4}	5.8×10^{-10}	2.6×10^{-5}	5.8×10^{-10}	8.9×10^{-4}	5.8×10^{-10}
	RMS error in k_z	2.35×10^{-4}		2.01×10^{-4}		1.8×10^{-4}		2.0×10^{-4}		1.5×10^{-4}		1.2×10^{-4}		8.2×10^{-5}	

B_1 - Random, B_2 - Constant: zero error in A-D converter.
C_1 - Random, C_2 - Constant: scale error in A-D convertor.
D_1 - Random, D_2 - Constant: ground instrument error.
E_1 - Random, E_2 - Constant: television camera error.

F_2 - Acc. misalignment error.

T = seconds of sampling period.

τ = sampling interval.

TABLE 1b

Components of error in k_y

Period 10 secs

τ secs	T secs	10		30		50		70		90		110		130	
		weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2
1.0	B_1	9.2×10^{-1}	1×10^{-6}	3.5×10^{-1}	1×10^{-6}	2.1×10^{-1}	1×10^{-6}	2.2×10^{-1}	1×10^{-6}	4.4×10^{-2}	1×10^{-6}	1.1×10^{-2}	1×10^{-6}	3.4×10^{-3}	1×10^{-6}
	B_2	6.8	2.2×10^{-5}	2.6	2.2×10^{-5}	1.5	2.2×10^{-5}	1.6	2.2×10^{-5}	3.2×10^{-1}	2.4×10^{-5}	8.2×10^{-2}	2.6×10^{-5}	2.5×10^{-2}	2.9×10^{-5}
	C_1	1.4×10^{-1}		1.4×10^{-1}		1.4×10^{-1}		1.4×10^{-1}		1.4×10^{-1}		1.4×10^{-1}		1.4×10^{-1}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	3.2×10^{-2}	28	1.2×10^{-2}	28	7.2×10^{-3}	28	7.5×10^{-3}	28	1.5×10^{-3}	28	3.8×10^{-4}	28	1.2×10^{-4}	28
	D_2	0		0		0		0		0		0		0	
	E_1	3.3×10^{3}	1.68×10^{-7}	1.7×10^{3}	1.68×10^{-7}	1.2×10^{3}	1.7×10^{-7}	1.2×10^{3}	1.68×10^{-7}	5.6×10^{2}	1.68×10^{-7}	3.6×10^{2}	1.68×10^{-7}	2.9×10^{2}	1.68×10^{-7}
	E_2	1.2×10^{4}		6.3×10^{3}		4.6×10^{3}		4.6×10^{3}		2.1×10^{3}		1.4×10^{3}		1.1×10^{3}	
	F_2	1.2×10^{4}	5.8×10^{-10}	6.3×10^{3}	5.8×10^{-10}	4.6×10^{3}	5.8×10^{-10}	4.6×10^{3}	5.8×10^{-10}	2.1×10^{3}	5.8×10^{-10}	1.4×10^{3}	5.8×10^{-10}	1.1×10^{3}	5.8×10^{-10}
	RMS error in k_y	9.5×10^{-1}		5.8×10^{-1}		4.5×10^{-1}		4.6×10^{-1}		2.1×10^{-1}		1.0×10^{-1}		5.8×10^{-2}	
0.5	B_1	4.7×10^{-1}	1×10^{-6}	1.8×10^{-1}	1×10^{-6}	1.1×10^{-1}	1×10^{-6}	1.1×10^{-1}	1×10^{-6}	2.2×10^{-2}	1×10^{-6}	5.6×10^{-3}	1×10^{-6}	1.7×10^{-3}	1×10^{-6}
	B_2	6.8	2.2×10^{-5}	2.6	2.2×10^{-5}	1.6	2.2×10^{-5}	1.6	2.2×10^{-5}	3.2×10^{-1}	2.4×10^{-5}	8.2×10^{-2}	2.6×10^{-5}	2.5×10^{-2}	2.9×10^{-5}
	C_1	6.9×10^{-2}		6.9×10^{-2}		6.9×10^{-2}		6.9×10^{-2}		6.9×10^{-2}		6.9×10^{-2}		6.9×10^{-2}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	1.2×10^{-3}	28	4.7×10^{-4}	28	2.8×10^{-4}	28	2.9×10^{-4}	28	5.8×10^{-5}	28	1.5×10^{-5}	28	4.5×10^{-6}	28
	D_2	0		0		0		0		0		0		0	
	E_1	1.7×10^{3}	1.68×10^{-7}	8.3×10^{2}	1.68×10^{-7}	6.4×10^{2}	1.68×10^{-7}	6.2×10^{3}	1.68×10^{-7}	3.0×10^{2}	1.68×10^{-7}	1.8×10^{2}	1.68×10^{-7}	1.5×10^{2}	1.68×10^{-7}
	E_2	1.3×10^{4}		6.4×10^{3}		4.7×10^{3}		4.7×10^{3}		2.2×10^{3}		1.4×10^{3}		1.1×10^{3}	
	F_2	1.3×10^{4}	5.8×10^{-10}	6.4×10^{3}	5.8×10^{-10}	4.7×10^{3}	5.8×10^{-10}	4.7×10^{3}	5.8×10^{-10}	2.2×10^{3}	5.8×10^{-10}	1.4×10^{3}	5.8×10^{-10}	1.1×10^{3}	5.8×10^{-10}
	RMS error in k_y	1.8×10^{-1}		1.2×10^{-1}		8.9×10^{-2}		9.6×10^{-2}		4.2×10^{-2}		2.1×10^{-2}		1.2×10^{-2}	

B_1 - Random, B_2 - Constant: zero error inaccuracy. C_1 - Random, C_2 - Constant: scale error inaccuracy. D_1 - Random, D_2 - Constant: ground instrument error. E_1 - Random, E_2 - Constant: television camera error.

F_2 - Acc. misalignment error.

T = seconds of sampling period.

τ = sampling interval.

TABLE 1b (Cont'd)

$\ddot{Y}$ Period 20 secs

τ secs	T secs	10 weight	10 σ^2	30 weight	30 σ^2	50 weight	50 σ^2	70 weight	70 σ^2	90 weight	90 σ^2	110 weight	110 σ^2	130 weight	130 σ^2
1.0	B_1	4.7×10^{-1}	1×10^{-6}	1.8×10^{-1}	1×10^{-6}	1.1×10^{-1}	1×10^{-6}	1.1×10^{-1}	1×10^{-6}	2.2×10^{-2}	1×10^{-6}	5.6×10^{-3}	1×10^{-6}	1.7×10^{-3}	1×10^{-6}
	B_2	6.8	2.2×10^{-5}	2.6	2.2×10^{-5}	1.6	2.2×10^{-5}	1.6	2.2×10^{-5}	3.2×10^{-1}	2.4×10^{-5}	8.1×10^{-2}	2.6×10^{-5}	2.5×10^{-2}	2.9×10^{-5}
	C_1	6.9×10^{-2}		6.9×10^{-2}		6.9×10^{-2}		6.9×10^{-2}		7.0×10^{-2}		7.0×10^{-2}		7.0×10^{-2}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	1.2×10^{-3}	28	4.7×10^{-4}	28	2.8×10^{-4}	28	2.9×10^{-4}	28	5.8×10^{-5}	28	1.5×10^{-5}	28	4.5×10^{-6}	28
	D_2	0		0		0		0		0		0		0	
	E_1	1.7×10^{3}	1.68×10^{-7}	8.9×10^{2}	1.68×10^{-7}	6.4×10^{2}	1.68×10^{-7}	6.5×10^{2}	1.68×10^{-7}	3.0×10^{2}	1.68×10^{-7}	1.9×10^{2}	1.68×10^{-7}	1.5×10^{2}	1.68×10^{-7}
	E_2	1.3×10^{4}		6.5×10^{3}		4.7×10^{3}		4.8×10^{3}		2.2×10^{3}		1.4×10^{3}		1.1×10^{3}	
	F_2	1.3×10^{4}	5.8×10^{-10}	6.5×10^{3}	5.8×10^{-10}	4.7×10^{3}	5.8×10^{-10}	4.8×10^{3}	5.8×10^{-10}	2.2×10^{3}	5.8×10^{-10}	1.4×10^{3}	5.8×10^{-10}	1.1×10^{3}	5.8×10^{-10}
	RMS error in k_y	1.9×10^{-1}		1.2×10^{-1}		9.0×10^{-2}		9.1×10^{-1}		4.1×10^{-2}		2.1×10^{-2}		1.2×10^{-2}	
0.5	B_1	2.4×10^{-1}	1×10^{-6}	9.2×10^{-2}	1×10^{-6}	5.4×10^{-2}	1×10^{-6}	5.6×10^{-2}	1×10^{-6}	1.1×10^{-2}	1×10^{-6}	2.8×10^{-3}	1×10^{-6}	8.8×10^{-4}	1×10^{-6}
	B_2	6.8	2.2×10^{-5}	2.6	2.2×10^{-5}	1.6	2.2×10^{-5}	1.6	2.2×10^{-5}	3.2×10^{-1}	2.4×10^{-5}	8.1×10^{-2}	2.6×10^{-5}	2.5×10^{-2}	2.9×10^{-5}
	C_1	3.5×10^{-2}		3.5×10^{-2}		3.5×10^{-2}		3.5×10^{-2}		3.5×10^{-2}		3.5×10^{-2}		3.5×10^{-2}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	4.3×10^{-5}	28	1.6×10^{-5}	28	9.7×10^{-6}	28	1.0×10^{-5}	28	2.0×10^{-6}	28	5.1×10^{-7}	28	1.6×10^{-7}	28
	D_2	0		0		0		0		0		0		0	
	E_1	8.9×10^{2}	1.68×10^{-7}	4.4×10^{2}	1.68×10^{-7}	3.3×10^{2}	1.68×10^{-7}	3.3×10^{2}	1.68×10^{-7}	1.5×10^{2}	1.68×10^{-7}	9.6×10^{1}	1.68×10^{-7}	7.9×10^{1}	1.68×10^{-7}
	E_2	1.3×10^{4}		6.5×10^{3}		4.7×10^{3}		4.8×10^{3}		2.2×10^{3}		1.4×10^{3}		1.1×10^{3}	
	F_2	1.3×10^{4}	5.8×10^{-10}	6.5×10^{3}	5.8×10^{-10}	4.7×10^{3}	5.8×10^{-10}	4.8×10^{3}	5.8×10^{-10}	2.2×10^{3}	5.8×10^{-10}	1.4×10^{3}	5.8×10^{-10}	1.1×10^{3}	5.8×10^{-10}
	RMS error in k_y	3.9×10^{-2}		2.4×10^{-2}		1.9×10^{-2}		1.9×10^{-2}		9.5×10^{-3}		5.8×10^{-3}		4.4×10^{-3}	

B_1 - Random, B_2 - Constant } zero error inaccuracy. C_1 - Random, C_2 - Constant } scale error inaccuracy. D_1 - Random, D_2 - Constant } ground instrument error. E_1 - Random, E_2 - Constant } television camera error.

F_2 - Acc. misalignment error.

T = seconds of sampling period.

τ = sampling interval.

TABLE 1b (Cont'd)

Period 30 secs

τ secs	T secs	10		30		50		70		90		110		130	
		weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2
1.0	B_1	3.1×10^{-1}	1×10^{-6}	1.2×10^{-1}	1×10^{-6}	7.2×10^{-2}	1×10^{-6}	7.4×10^{-2}	1×10^{-6}	1.5×10^{-2}	1×10^{-6}	3.7×10^{-3}	1×10^{-6}	1.2×10^{-3}	1×10^{-6}
	B_2	6.8	2.2×10^{-5}	2.6	2.2×10^{-5}	1.6	2.2×10^{-5}	1.6	2.2×10^{-5}	3.2×10^{-1}	2.4×10^{-5}	8.1×10^{-2}	2.6×10^{-5}	2.5×10^{-2}	2.9×10^{-5}
	C_1	4.7×10^{-2}		4.7×10^{-2}		4.6×10^{-2}		4.7×10^{-2}		4.8×10^{-2}		4.8×10^{-2}		4.8×10^{-2}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	1.7×10^{-4}	28	6.6×10^{-5}	28	3.9×10^{-5}	28	4.1×10^{-5}	28	8.1×10^{-6}	28	2.1×10^{-6}	28	6.4×10^{-7}	28
	D_2	0		0		0		0		0		0		0	
	E_1	1.2×10^{3}	1.68×10^{-7}	6.0×10^{2}	1.68×10^{-7}	4.3×10^{2}	1.68×10^{-7}	4.4×10^{2}	1.68×10^{-7}	2.0×10^{2}	1.68×10^{-7}	1.3×10^{2}	1.68×10^{-7}	1.1×10^{2}	1.68×10^{-7}
	E_2	1.3×10^{4}		6.5×10^{3}		4.7×10^{3}		4.8×10^{3}		2.2×10^{3}		1.4×10^{3}		1.1×10^{3}	
	F_2	1.3×10^{4}	5.8×10^{-10}	6.5×10^{3}	5.8×10^{-10}	4.7×10^{3}	5.8×10^{-10}	4.8×10^{3}	5.8×10^{-10}	2.2×10^{3}	5.8×10^{-10}	1.4×10^{3}	5.8×10^{-10}	1.1×10^{3}	5.8×10^{-10}
	RMS error in k_y	7.2×10^{-2}		4.5×10^{-2}		3.5×10^{-2}		3.5×10^{-2}		1.6×10^{-2}		9.1×10^{-3}		6.1×10^{-3}	
0.5	B_1	1.6×10^{-1}	1×10^{-6}	6.1×10^{-2}	1×10^{-6}	3.6×10^{-2}	1×10^{-6}	3.8×10^{-2}	1×10^{-6}	7.5×10^{-3}	1×10^{-6}	1.9×10^{-3}	1×10^{-6}	5.9×10^{-4}	1×10^{-6}
	B_2	6.8	2.2×10^{-5}	2.6	2.2×10^{-5}	1.6	2.2×10^{-5}	1.6	2.2×10^{-5}	3.2×10^{-1}	2.4×10^{-5}	8.1×10^{-2}	2.6×10^{-5}	2.5×10^{-2}	2.9×10^{-5}
	C_1	2.4×10^{-2}		2.4×10^{-2}		2.4×10^{-2}		2.4×10^{-2}		2.4×10^{-2}		2.4×10^{-2}		2.4×10^{-2}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	5.8×10^{-6}	28	2.2×10^{-6}	28	1.3×10^{-6}	28	1.4×10^{-6}	28	2.8×10^{-7}	28	6.9×10^{-8}	28	2.1×10^{-8}	28
	D_2	0		0		0		0		0		0		0	
	E_1	5.9×10^{2}	1.68×10^{-7}	3.0×10^{2}	1.68×10^{-7}	2.2×10^{2}	1.68×10^{-7}	2.2×10^{2}	1.68×10^{-7}	1.0×10^{2}	1.68×10^{-7}	6.6×10^{1}	1.68×10^{-7}	5.4×10^{1}	1.68×10^{-7}
	E_2	1.3×10^{4}		6.5×10^{3}		4.7×10^{3}		4.8×10^{3}		2.2×10^{3}		1.4×10^{3}		1.1×10^{3}	
	F_2	1.3×10^{4}	5.8×10^{-10}	6.5×10^{3}	5.8×10^{-10}	4.7×10^{3}	5.8×10^{-10}	4.8×10^{3}	5.8×10^{-10}	2.2×10^{3}	5.8×10^{-10}	1.4×10^{3}	5.8×10^{-10}	1.1×10^{3}	5.8×10^{-10}
	RMS error in k_y	2.0×10^{-2}		1.3×10^{-2}		1.1×10^{-2}		1.1×10^{-2}		5.8×10^{-3}		4.0×10^{-3}		3.3×10^{-3}	

B_1 - Random, B_2 - Constant: zero error inaccuracy. C_1 - Random, C_2 - Constant: scale error inaccuracy. D_1 - Random, D_2 - Constant: ground instrument error. E_1 - Random, E_2 - Constant: television camera error.

F_2 - Acc. misalignment error.

T = seconds of sampling period.

τ = sampling interval.

TABLE 1b (Cont'd)

Period 40 secs

τ secs \ T secs		10		30		50		70		90		110		130	
		weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2	weight	σ^2
1.0	B_1	2.4×10^{-1}	1×10^{-6}	9.1×10^{-2}	1×10^{-6}	5.4×10^{-2}	1×10^{-6}	5.6×10^{-2}	1×10^{-6}	1.1×10^{-2}	1×10^{-6}	2.8×10^{-3}	1×10^{-6}	8.6×10^{-4}	1×10^{-6}
	B_2	6.7	2.2×10^{-5}	2.6	2.2×10^{-5}	1.6	2.2×10^{-5}	1.6	2.2×10^{-5}	3.1×10^{-1}	2.4×10^{-5}	8.0×10^{-2}	2.6×10^{-5}	2.5×10^{-2}	2.9×10^{-5}
	C_1	3.6×10^{-2}		3.5×10^{-2}		3.5×10^{-2}		3.6×10^{-2}		3.7×10^{-2}		3.7×10^{-2}		3.7×10^{-2}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	4.2×10^{-5}	28	1.6×10^{-5}	28	9.7×10^{-6}	28	1.0×10^{-5}	28	2.0×10^{-6}	28	5.1×10^{-7}	28	1.6×10^{-7}	28
	D_2	0		0		0		0		0		0		0	
	E_1	8.8×10^{2}	1.68×10^{-7}	4.5×10^{2}	1.68×10^{-7}	3.3×10^{2}	1.68×10^{-7}	3.3×10^{2}	1.68×10^{-7}	1.6×10^{2}	1.68×10^{-7}	1.0×10^{2}	1.68×10^{-7}	8.1×10^{1}	1.68×10^{-7}
	E_2	1.3×10^{4}		6.5×10^{3}		4.7×10^{3}		4.8×10^{3}		2.2×10^{3}		1.4×10^{3}		1.1×10^{3}	
	F_2	1.3×10^{4}	5.8×10^{-10}	6.5×10^{3}	5.8×10^{-10}	4.7×10^{3}	5.8×10^{-10}	4.8×10^{3}	5.8×10^{-10}	2.2×10^{3}	5.8×10^{-10}	1.4×10^{3}	5.8×10^{-10}	1.1×10^{3}	5.8×10^{-10}
	RMS error in k_y	3.8×10^{-2}		2.4×10^{-2}		1.9×10^{-2}		1.9×10^{-2}		9.6×10^{-3}		5.8×10^{-3}		4.4×10^{-3}	
0.5	B_1	1.2×10^{-1}	1×10^{-6}	4.6×10^{-2}	1×10^{-6}	2.7×10^{-2}	1×10^{-6}	2.8×10^{-2}	1×10^{-6}	5.5×10^{-3}	1×10^{-6}	1.4×10^{-3}	1×10^{-6}	4.4×10^{-4}	1×10^{-6}
	B_2	6.7	2.2×10^{-5}	2.6	2.2×10^{-5}	1.6	2.2×10^{-5}	1.6	2.2×10^{-5}	3.1×10^{-1}	2.4×10^{-5}	8.0×10^{-2}	2.6×10^{-5}	2.5×10^{-2}	2.9×10^{-5}
	C_1	1.8×10^{-2}		1.8×10^{-2}		1.8×10^{-2}		1.8×10^{-2}		1.9×10^{-2}		1.9×10^{-2}		1.9×10^{-2}	
	C_2	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}	1	6.3×10^{-10}
	D_1	1.4×10^{-6}	28	5.4×10^{-7}	28	3.2×10^{-7}	28	3.3×10^{-7}	28	6.6×10^{-8}	28	1.7×10^{-8}	28	5.2×10^{-9}	28
	D_2	0		0		0		0		0		0		0	
	E_1	4.5×10^{2}	1.68×10^{-7}	2.3×10^{2}	1.68×10^{-7}	1.7×10^{2}	1.68×10^{-7}	1.7×10^{2}	1.68×10^{-7}	7.8×10^{1}	1.68×10^{-7}	5.0×10^{1}	1.68×10^{-7}	4.1×10^{1}	1.68×10^{-7}
	E_2	1.3×10^{4}		6.5×10^{3}		4.7×10^{3}		4.8×10^{3}		2.2×10^{3}		1.4×10^{3}		1.1×10^{3}	
	F_2	1.3×10^{4}	5.8×10^{-10}	6.5×10^{3}	5.8×10^{-10}	4.7×10^{3}	5.8×10^{-10}	4.8×10^{3}	5.8×10^{-10}	2.2×10^{3}	5.8×10^{-10}	1.4×10^{3}	5.8×10^{-10}	1.1×10^{3}	5.8×10^{-10}
	RMS error in k_y	1.6×10^{-2}		1.1×10^{-2}		8.7×10^{-3}		8.7×10^{-3}		4.9×10^{-3}		3.4×10^{-3}		2.9×10^{-3}	

B_1 - Random, B_2 - Constant } zero error inaccuracy.

C_1 - Random, C_2 - Constant } scale error inaccuracy.

D_1 - Random, D_2 - Constant } ground instrument error.

E_1 - Random, E_2 - Constant } television camera error.

F_2 - Acc. misalignment error.

T = seconds of sampling period.

τ = sampling interval.

TABLE 2a

Errors in k_{0z}, k_{1z}, k_{2z} in terms of σ^2_{kz}

Factors multiplying σ^2_{kz}

No. of sampling periods	5	10	20	28
k_{0z}	53749	18554	7752	5294
k_{1z}	44.27	12.89	4.90	3.263
k_{2z}	0.00157	0.00033	0.00011	0.00007
k_{0z}	5692	2547	1240	879
k_{1z}	1.85	0.7602	0.3488	0.2433

TABLE 2b

Errors in k_{0y}, k_{1y}, k_{2y} in terms of σ^2_{ky}

Factors multiplying σ^2_{ky}

No. of sampling periods	5	10	20	28
k_{0y}	0.36	0.229	0.130	0.0978
k_{1y}	2.53	1.16	0.558	0.396
k_{2y}	0.30	0.081	0.030	0.020
k_{0y}	0.16	0.10	0.066	0.45
k_{1y}	0.57	0.28	0.14	0.10

TABLE 3a

Components of error in k_{0z}, k_{1z}, k_{2z}

τ = 5.0 Period 0-140 secs

		k_0		k_1		k_0		k_1		k_2	
		ωt	σ^2	ωt	σ^2	ωt	σ^2	ωt	σ^2	ωt	σ^2
Random zero error in accelerometer	B_1	4.1×10^{-1}	6.8×10^{-6}	8.01×10^{-5}	6.8×10^{-6}	3.3×10^{0}	6.8×10^{-6}	1.9×10^{-3}	6.8×10^{-6}	4.2×10^{-8}	6.8×10^{-6}
Constant zero error in accelerometer	B_2	1	1.8×10^{-5}	0	1.8×10^{-5}	1	1.8×10^{-5}	0	1.8×10^{-5}	0	1.8×10^{-5}
Random scale error in accelerometer	C_1	1.3×10^{3}		3.6×10^{-1}		8.6×10^{3}		5.5×10^{0}		1.3×10^{-4}	
Constant scale error in accelerometer	C_2	0	6.3×10^{-10}	1	6.3×10^{-10}	0	6.3×10^{-10}	1	6.3×10^{-10}	0	6.3×10^{-10}
Random ground instrument error (weighted)	D_1	1.6×10^{-5}	1	5.6×10^{-9}	1	4.0×10^{-4}	1	3.3×10^{-7}	1	1.1×10^{-11}	1
Constant ground instrument error (weighted)	D_2	2.0×10^{-6}	1	3.7×10^{-10}	1	8.0×10^{-5}	1	3.0×10^{-8}	1	7.8×10^{-13}	1
Random television camera error	E_1	4.3×10^{-1}	1.7×10^{-7}	1.7×10^{-4}	1.7×10^{-7}	2.0×10^{0}	1.7×10^{-7}	1.5×10^{-3}	1.7×10^{-7}	4.2×10^{-8}	1.7×10^{-7}
Constant television camera error	E_2	1.3×10^{0}		1.2×10^{-3}		1.3×10^{0}		1.2×10^{-3}		1.1×10^{-11}	
accelerometer misalignment error	F_2	1.3×10^{0}	5.8×10^{-10}	1.2×10^{-3}	5.8×10^{-10}	1.3×10^{0}	5.8×10^{-10}	1.2×10^{-3}	5.8×10^{-10}	1.1×10^{-11}	5.8×10^{-10}
Total R.M.S. error		$6.2\times10^{-3} = 1.9\times10^{-4}g$		8.4×10^{-5}		$2.2\times10^{-2} = 6.9\times10^{-4}g$		6.2×10^{-4}		3.5×10^{-6}	
Ground instrumentation, equations weighted	D_1	1.5×10^{-5}	1	5.1×10^{-9}	1	3.6×10^{-4}	1	2.9×10^{-7}	1	1.0×10^{-11}	1

τ = 1.0 Period 0-138 secs

		k_0		k_1		k_0		k_1		k_2	
		ωt	σ^2	ωt	σ^2	ωt	σ^2	ωt	σ^2	ωt	σ^2
Random zero error in accelerometer	B_1	1.0×10^{-1}	6.8×10^{-6}	2.1×10^{-5}	6.8×10^{-6}	9.3×10^{-1}	6.8×10^{-6}	5.9×10^{-4}	6.8×10^{-6}	1.5×10^{-6}	6.8×10^{-6}
Constant zero error in accelerometer	B_2	1	1.8×10^{-5}	0	1.8×10^{-5}	1	1.8×10^{-5}	0	1.8×10^{-5}	0	1.8×10^{-5}
Random scale error in accelerometer	C_1	3.0×10^{2}		8.5×10^{-2}		2.3×10^{3}		1.6×10^{0}		4.2×10^{-5}	
Constant scale error in accelerometer	C_2	0	6.3×10^{-10}	1	6.3×10^{-10}	0	6.3×1^{-10}	1	6.3×10^{-10}	0	6.3×10^{-10}
Random ground instrument error (weighted)	D_1	4.2×10^{-6}	1	1.5×10^{-9}	1	1.1×10^{-4}	1	9.8×10^{-8}	1	3.7×10^{-12}	1
Constant ground instrument error (weighted)	D_2	2.0×10^{-6}	1	3.7×10^{-10}	1	4.7×10^{-5}	1	3.0×10^{-8}	1	7.8×10^{-13}	1
Random television camera error	E_1	9.1×10^{-2}	1.7×10^{-7}	3.5×10^{-5}	1.7×10^{-7}	4.7×10^{-1}	1.7×10^{-7}	3.6×10^{-4}	1.7×10^{-7}	1.2×10^{-8}	1.7×10^{-7}
Constant television camera error	E_2	1.3×10^{0}		1.2×10^{-3}		1.3×10^{0}		1.3×10^{-3}		0	
accelerometer misalignment error	F_2	1.3×10^{0}	5.8×10^{-10}	1.2×10^{-3}	5.8×10^{-10}	1.3×10^{0}	5.8×10^{-10}	1.3×10^{-3}	5.8×10^{-10}	2.0×10^{-12}	5.8×10^{-10}
Total R.M.S. error		$5.0\times10^{-3} = 1.6\times10^{-4}g$		5.2×10^{-5}		$1.3\times10^{-2} = 4.1\times10^{-4}g$		3.6×10^{-4}		2.1×10^{-6}	
Ground instrumentation, equations weighted	D_1	3.8×10^{-6}	1	1.4×10^{-9}	1	1.0×10^{-4}	1	3.8×10^{-8}	1	3.4×10^{-12}	1

TABLE 3a (Cont'd)

$\tau = 0.2$ Period 0–140 secs

		k_0		k_1		k_0		k_1		k_2	
		ωt	σ^2	ωt	σ^2	ωt	σ^2	ωt	σ^2	ωt	σ^2
Random } zero error in accelerometer	B_1	1.6×10^{-2}	6.8×10^{-6}	3.2×10^{-6}	6.8×10^{-6}	1.3×10^{-1}	6.8×10^{-6}	7.6×10^{-5}	6.8×10^{-6}	1.7×10^{-9}	6.8×10^{-6}
Constant } zero error in accelerometer	B_2	1	1.8×10^{-5}	0	1.8×10^{-5}	1	1.8×10^{-5}	0	1.8×10^{-5}	0	1.8×10^{-5}
Random } scale error in accelerometer	C_1	5.2×10^{1}		1.4×10^{-2}		3.4×10^{2}		2.2×10^{-1}		5.2×10^{-6}	
Constant } scale error in accelerometer	C_2	0	6.3×10^{-10}	1	6.3×10^{-10}	0	6.3×10^{-10}	1	6.3×10^{-10}	0	6.3×10^{-10}
Random } ground instrument error (weighted)	D_1	6.4×10^{-7}	1	2.2×10^{-10}	1	1.6×10^{-5}	1	1.3×10^{-8}	1	4.4×10^{-13}	1
Constant } ground instrument error (weighted)	D_2	2.0×10^{-6}	1	3.7×10^{-10}	1	4.7×10^{-5}	1	3.0×10^{-8}	1	7.8×10^{-13}	1
Random } television camera error	E_1	9.1×10^{-2}	1.7×10^{-7}	3.5×10^{-5}	1.7×10^{-7}	4.7×10^{-1}	1.7×10^{-7}	3.6×10^{-4}	1.7×10^{-7}	1.2×10^{-8}	1.7×10^{-7}
Constant } television camera error	E_2	1.3×10^{0}		1.2×10^{-3}		1.3×10^{0}		1.3×10^{-3}		0	
Accelerometer misalignment error	F_2	1.3×10^{0}	5.8×10^{-10}	1.2×10^{-3}	5.8×10^{-10}	1.3×10^{0}	5.8×10^{-10}	1.3×10^{-3}	5.8×10^{-10}	2.0×10^{-12}	5.8×10^{-10}
Total R.M.S. error		$4.6\times10^{-3} = 1.4\times10^{-4}g$		3.5×10^{-5}		$9.0\times10^{-3} = 2.8\times10^{-4}g$		2.1×10^{-4}		1.1×10^{-6}	
Ground instrument, equations weighted	D_1	6.0×10^{-7}	1	2.0×10^{-10}	1	1.4×10^{-5}	1	1.2×10^{-8}	1	4.0×10^{-13}	1

TABLE 3b

Components of error in k_{0y}, k_{1y}, k_{2y}

$\tau = 5.0$ Period 0-140 secs

		k_0		k_1		k_0		k_1		k_2	
		ωt	σ^2	ωt	σ^2	ωt	σ^2	ωt	σ^2	ωt	σ^2
Random zero error in accelerometer	B_1	1.4×10^{-1}	6.8×10^{-6}	6.4×10^{-2}	6.8×10^{-6}	4.1×10^{-1}	6.8×10^{-6}	8.0×10^{-1}	6.8×10^{-6}	3.0×10^{-2}	6.8×10^{-6}
Constant zero error in accelerometer	B_2	1	1.8×10^{-5}	0	1.8×10^{-5}	1	1.8×10^{-5}	0	1.8×10^{-5}	0	1.8×10^{-5}
Random scale error in accelerometer	C_1	7.1×10^{-2}		1.3×10^{-1}		1.7×10^{-1}		5.8×10^{-1}		2.9×10^{-2}	
Constant scale error in accelerometer	C_2	0	6.3×10^{-10}	1	6.3×10^{-10}	0	6.3×10^{-10}	1	6.8×10^{-10}	0	6.3×10^{-10}
Random ground instrument error	D_1	5.0×10^{-8}	25	7.7×10^{-8}	25	5.6×10^{-7}	25	2.1×10^{-6}	25	1.3×10^{-7}	25
Constant ground instrument error	D_2	3.4×10^{-7}	2.1	2.8×10^{-7}	2.1	2.9×10^{-6}	2.1	7.5×10^{-6}	2.1	3.2×10^{-7}	2.1
Random television camera error	E_1	3.42×10^{2}	1.7×10^{-7}	2.9×10^{2}	1.7×10^{-7}	9.5×10^{2}	1.7×10^{-7}	2.3×10^{3}	1.7×10^{-7}	9.1×10^{1}	1.7×10^{-7}
Constant television camera error	E_2	1.0×10^{3}		8.1×10^{2}		1.0×10^{3}		8.0×10^{2}		6.8×10^{-4}	
Accelerometer misalignment error	F_2	1.0×10^{3}	5.8×10^{-10}	8.1×10^{2}	5.8×10^{-10}	1.0×10^{3}	5.8×10^{-10}	8.0×10^{2}	5.8×10^{-10}	6.8×10^{-4}	5.8×10^{-10}
Total R.M.S. error		8.9×10^{-3} fps^2		7.2×10^{-3}		1.4×10^{-2} fps^2		2.1×10^{-2}		4.4×10^{-3}	

$\tau = 1.0$ Period 0-138 secs

		k_0		k_1		k_0		k_1		k_2	
		ωt	σ^2	ωt	σ^2	ωt	σ^2	ωt	σ^2	ωt	σ^2
Random zero error in accelerometer	B_1	3.2×10^{-2}	6.8×10^{-6}	1.7×10^{-2}	6.8×10^{-2}	1.0×10^{-1}	6.8×10^{-6}	2.2×10^{-1}	6.8×10^{-6}	9.7×10^{-3}	6.8×10^{-6}
Constant zero error in accelerometer	B_2	1	1.8×10^{-5}	0	1.8×10^{-5}	1	1.8×10^{-5}	0	1.8×10^{-5}	0	1.8×10^{-5}
Random scale error in accelerometer	C_1	1.5×10^{-2}		2.8×10^{-2}		3.6×10^{-2}		1.3×10^{-1}		7.5×10^{-3}	
Constant scale error in accelerometer	C_2	0	6.3×10^{-10}	1	6.3×10^{-10}	0	6.3×10^{-10}	1	6.3×10^{-10}	0	6.3×10^{-10}
Random ground instrument error	D_1	1.3×10^{-8}	25	2.2×10^{-8}	25	1.5×10^{-7}	25	6.1×10^{-7}	25	4.6×10^{-8}	25
Constant ground instrument error	D_2	3.4×10^{-7}	2.1	2.8×10^{-7}	2.1	2.9×10^{-6}	2.1	7.5×10^{-6}	2.1	3.2×10^{-7}	2.1
Random television camera error	E_1	7.6×10^{1}	1.7×10^{-7}	6.9×10^{1}	1.7×10^{-7}	2.3×10^{2}	1.7×10^{-7}	5.8×10^{2}	1.7×10^{-7}	2.8×10^{1}	1.7×10^{-7}
Constant television camera error	E_2	1.0×10^{3}		8.1×10^{2}		1.0×10^{3}		8.0×10^{2}		9.4×10^{-4}	
Accelerometer misalignment error	F_2	1.0×10^{3}	5.8×10^{-10}	8.1×10^{2}	5.8×10^{-10}	1.0×10^{3}	5.8×10^{-10}	8.0×10^{2}	5.8×10^{-10}	9.4×10^{-4}	5.8×10^{-10}
Total R.M.S. error		5.7×10^{-3}		3.6×10^{-3}		8.2×10^{-3} fps^2		1.1×10^{-2}		2.6×10^{-3}	

TABLE 3b (Cont'd)

$\tau = 0.2$ Period 0-140 secs

		k_0		k_1		k_0		k_1		k_2	
		ωt	σ^2	ωt	σ^2	ωt	σ^2	ωt	σ^2	ωt	σ^2
Random } zero error in accelerometer	B_1	5.6×10^{-3}	6.8×10^{-6}	2.6×10^{-3}	6.8×10^{-6}	1.6×10^{-2}	6.8×10^{-6}	3.2×10^{-2}	6.8×10^{-6}	1.2×10^{-3}	6.8×10^{-6}
Constant } zero error in accelerometer	B_2	1	1.8×10^{-5}	0	1.8×10^{-5}	1	1.8×10^{-5}	0	1.8×10^{-5}	0	1.8×10^{-5}
Random } scale error in accelerometer	C_1	2.9×10^{-3}		5.3×10^{-3}		6.7×10^{-3}		2.3×10^{-2}		1.2×10^{-3}	
Constant } scale error in accelerometer	C_2	0	6.3×10^{-10}	1	6.3×10^{-10}	0	6.3×10^{-10}	1	6.3×10^{-10}	0	6.3×10^{-10}
Random } ground instrument error	D_1	2.0×10^{-9}	25	3.1×10^{-9}	25	2.3×10^{-8}	25	8.4×10^{-8}	25	5.3×10^{-9}	25
Constant } ground instrument error	D_2	3.4×10^{-7}	2.1	2.8×10^{-7}	2.1	2.9×10^{-6}	2.1	7.5×10^{-6}	2.1	3.2×10^{-7}	2.1
Random } television camera error	E_1	7.6×10^{1}	1.7×10^{-7}	6.9×10^{1}	1.7×10^{-7}	2.25×10^{2}	1.7×10^{-7}	5.8×10^{2}	1.7×10^{-7}	2.8×10^{1}	1.7×10^{-7}
Constant } television camera error	E_2	1.0×10^{3}		8.1×10^{2}		1.0×10^{3}		8.0×10^{2}		9.4×10^{-4}	
Accelerometer misalignment error	F_2	1.0×10^{3}	5.8×10^{-10}	8.1×10^{2}	5.8×10^{-10}	1.0×10^{3}	5.8×10^{-10}	8.0×10^{2}	5.8×10^{-10}	9.4×10^{-4}	5.8×10^{-10}
Total R.M.S. error		5.7×10^{-3}		3.6×10^{-3}		8.1×10^{-3}		1.1×10^{-2}		2.4×10^{-3}	

TABLE 4

Errors due to ignoring k_2

Z

	k_{Oz}		k_{1z}	
	ωt	$\sigma_{k_2}^2$	ωt	$\sigma_{k_2}^2$
τ = 5 secs	3.8×10^7	4×10^{-14}	3.1×10^4	4×10^{-14}
τ = 1 sec	3.3×10^7	4×10^{-14}	2.8×10^4	4×10^{-14}
approx. r.m.s. error	1.2×10^{-3} fps^2		3.4×10^{-5}	

Y

	k_{Oz}		k_{1z}	
	ωt	$\sigma_{k_2}^2$	ωt	$\sigma_{k_2}^2$
τ = 5 secs	3.8×10^0	4×10^{-4}	1.5×10^1	4×10^{-4}
τ = 1 sec	3.0×10^0	4×10^{-4}	1.3×10^1	4×10^{-4}
approx. r.m.s. error	3.7×10^{-7} fps^2		7.5×10^{-7}	

TABLE 5

Covariance matrices for R,θ,ϕ calculated using ballistic cameras

(Provided by Mr. R.H. Gooding)

$$\begin{bmatrix} \sigma_{Rc}^2 & \sigma_{Rc}\sigma_{\theta c} & \sigma_{Rc}\sigma_{\phi c} \\ \sigma_{Rc}\sigma_{\theta c} & \sigma_{\theta c}^2 & \sigma_{\theta c}\sigma_{\phi c} \\ \sigma_{Rc}\sigma_{\phi c} & \sigma_{\theta c}\sigma_{\phi c} & \sigma_{\phi c}^2 \end{bmatrix}_t$$

R is measured in feet and θ and ϕ in mils.

t = 16.5 secs
$$\begin{bmatrix} 10.1 & -0.00436 & 0.0526 \\ -0.00436 & 0.000372 & -0.000002 \\ 0.0526 & -0.000002 & 0.00152 \end{bmatrix}$$

t = 82 secs
$$\begin{bmatrix} 10.1 & -0.0105 & 0.0366 \\ -0.0105 & 0.000395 & -0.000026 \\ 0.0366 & -0.000026 & 0.00123 \end{bmatrix}$$

t = 113.5 secs
$$\begin{bmatrix} 14.3 & 0.0089 & 0.0144 \\ 0.0089 & 0.000438 & -0.000045 \\ 0.0144 & -0.000045 & 0.00104 \end{bmatrix}$$

t = 146 secs
$$\begin{bmatrix} 129 & 0.132 & -0.0305 \\ 0.132 & 0.000439 & -0.000006 \\ -0.0305 & -0.000006 & 0.00220 \end{bmatrix}$$

FIG. 1.

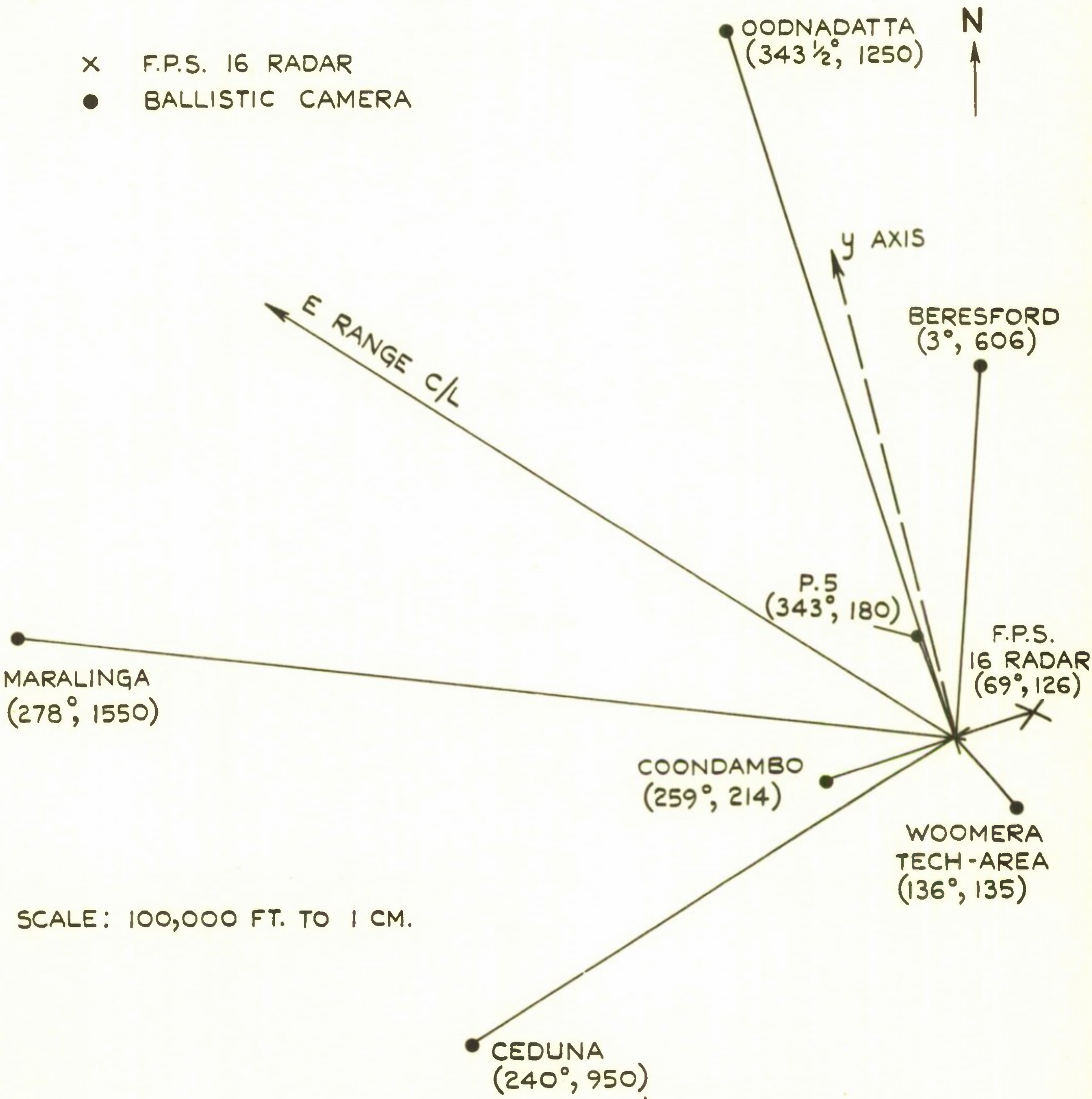

FIG. 1. POSITIONS OF GROUND INSTRUMENTATION.

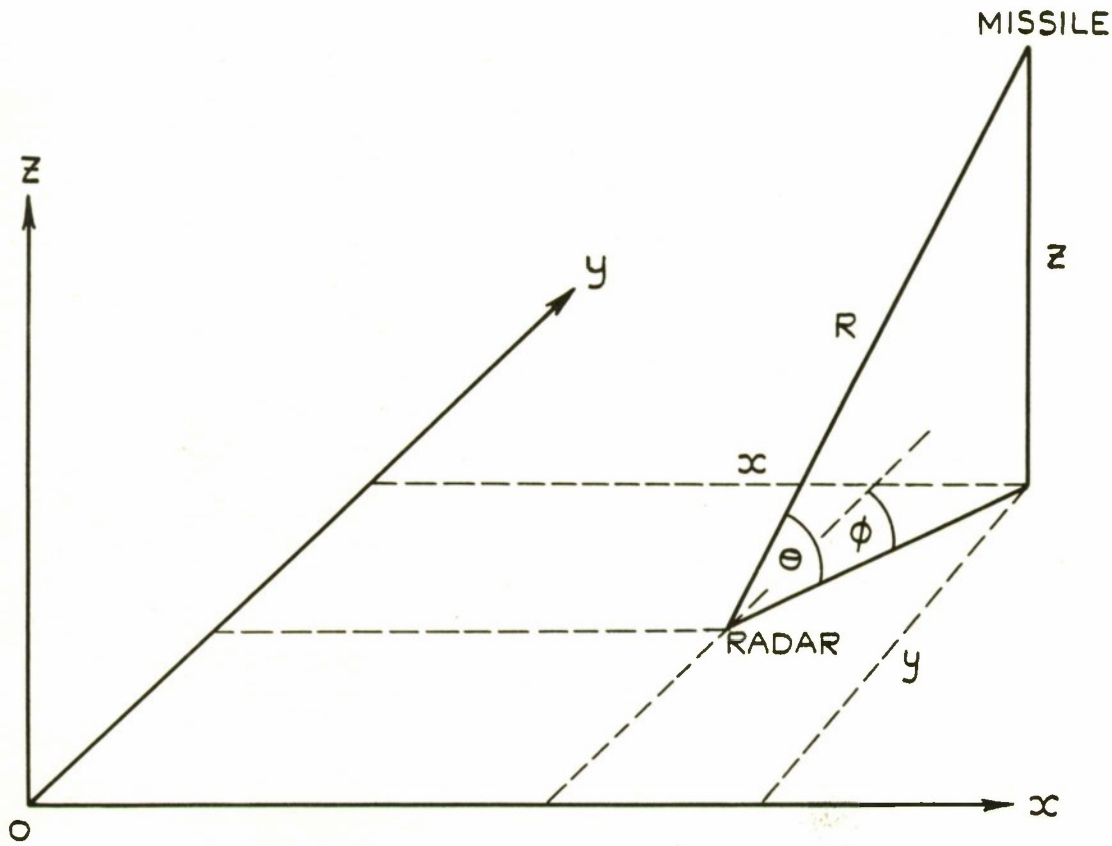

FIG. 2. RADAR CO-ORDINATES R, θ, φ.

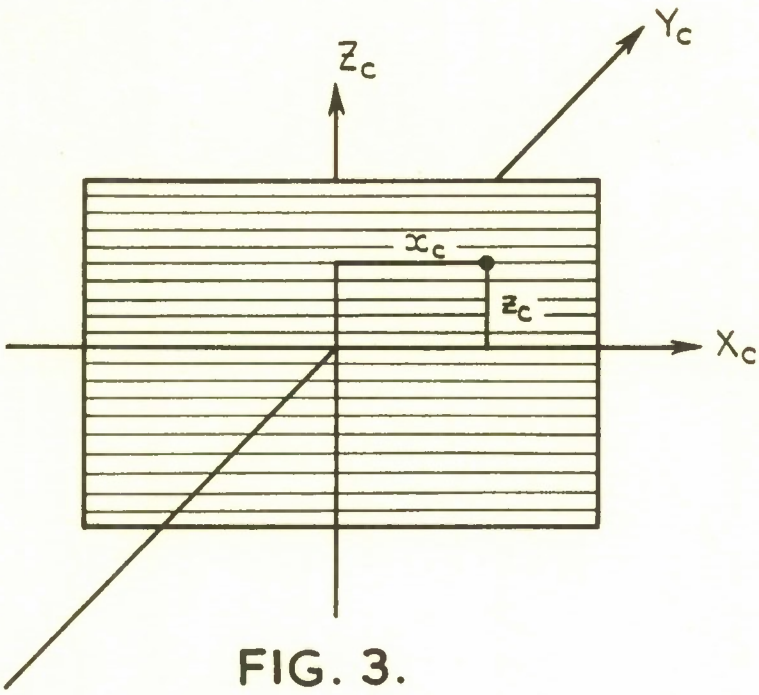

FIG. 3.

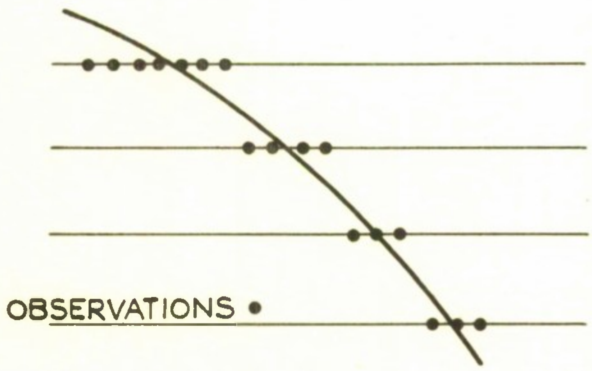

FIG. 4.

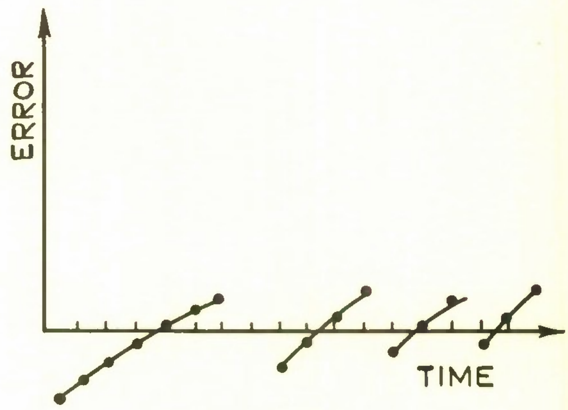

FIG. 5.

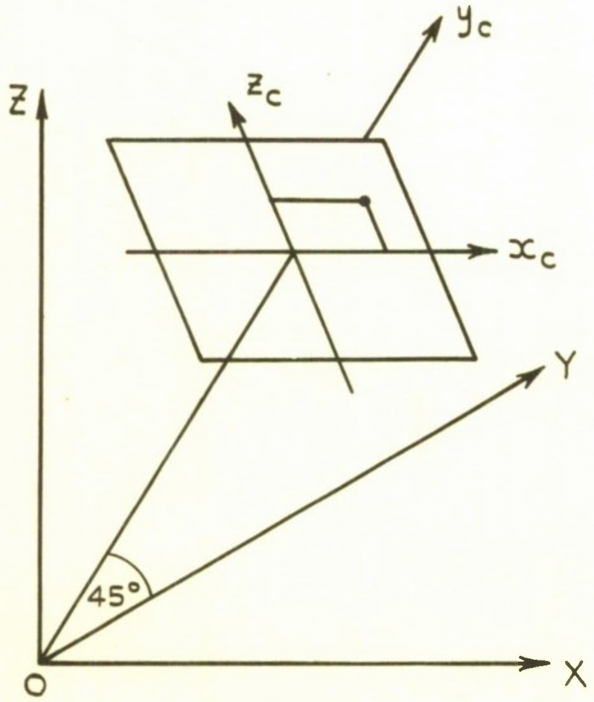

FIG. 6.

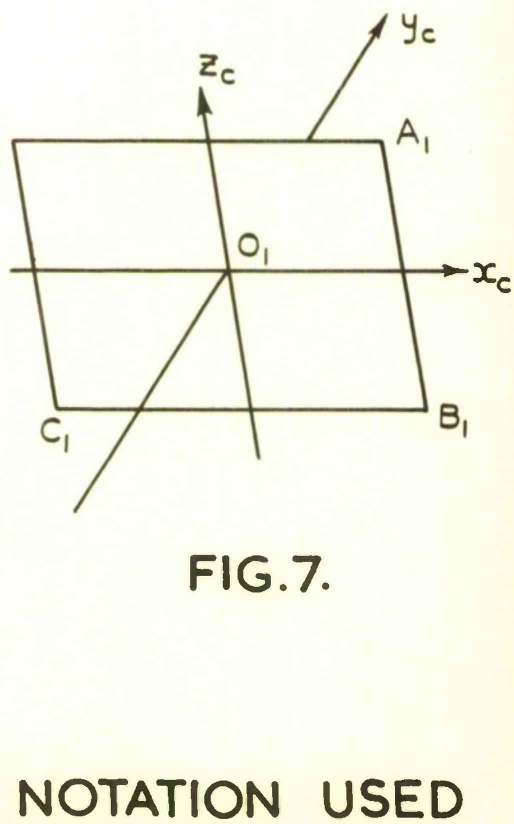

FIG. 7.

FIG. 3 - 7. DIAGRAMS SHOWING NOTATION USED IN TELEVISION CAMERA CALCULATIONS.

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